Economic Modelling 53 (2016) 419–435

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/ecmod

When Behavioral Portfolio Theory meets Markowitz theory☆,☆☆

Marie Pfiffelmann [a], Tristan Roger [b], Olga Bourachnikova [a]

[a] LaRGE Research Center, EM Strasbourg Business School, University of Strasbourg, France
[b] DRM Finance, Paris-Dauphine University, France

ARTICLE INFO

Article history:
Accepted 18 October 2015
Available online 28 November 2015

Keywords:
Behavioral Portfolio Theory
Mean Variance Theory
Portfolio optimization
Decision making

ABSTRACT

The Behavioral Portfolio Theory (BPT) developed by Shefrin and Statman (2000) is often set against Markowitz's (1952) Mean Variance Theory (MVT). In this paper, we compare the asset allocations generated by BPT and MVT without restrictions. Using U.S. stock prices from the CRSP database for the 1995–2011 period, this paper is the first study that empirically determines the BPT optimal portfolio. We show that Shefrin and Statman's (2000) optimal portfolio is Mean Variance (MV) efficient in more than 70% of cases. However, our results also indicate that the BPT portfolio exhibits a high level of risk, high returns and positively skewed returns. We show that the risk aversion coefficient of the BPT portfolio is up to 10 times lower than the risk aversion degree shown by typical MV investors. Even if the asset allocations may coincide, typical MV investors will not usually select the BPT optimal portfolios. These results underline that MVT and BPT cannot be used interchangeably.

1. Introduction

The characteristics of the optimal portfolio of investors is a key question for both academia and financial practitioners. For several decades, Markowitz's (1952) Mean Variance Theory (MVT) has been considered as the cornerstone of modern portfolio theory. In MVT, asset allocation by investors results from a trade-off between expected return and variance. The optimal portfolio is a perfectly diversified portfolio that exhibits the lowest risk for a given level of expected return. Since this formulation, a number of studies (Cox and Huang, 1989; Li and Ng, 2000; Merton, 1971; Yao et al., 2014) have extended Markowitz's (1952) framework by taking into account new elements such as market incompleteness, labor income and dynamic formulation of the model.[1] However, experimental evidence (Allais, 1953; Kahneman and Tversky, 1979) indicates that the standard expected utility, which is the key assumption of Markowitz's (1952) mean variance (MV) framework, fails to adequately explain investor behavior. For instance, most individuals purchasing insurance contracts to protect against unlikely events also carry out gambles where the probability of winning is extremely small. The coexistence of these risk averse and risk-seeking behaviors cannot be captured by the utility function. Similarly, Thaler (1985, 1999) raised the idea that individuals divide their current and future assets into separate, non-transferable portions. They create distinct mental accounts which are handled separately and differently. To better capture these different features, Shefrin and Statman (2000) developed an alternative model of portfolio choice, the Behavioral Portfolio Theory (BPT hereafter). The foundation of BPT is in sharp contradiction to the foundation of MVT. First, the risk in BPT relates to the downside risk rather than the variance of returns. BPT investors set a safety first constraint, as set out in Roy's (1952) model. They aim to secure their wealth in a maximum number of states of nature. Second, BPT integrates the fact that investors do not behave rationally. BPT assumes that two conflicting emotions (fear and hope) drive investor behavior (Lopes, 1987). In contrast with MVT, BPT therefore incorporates probability weighting, which allows for the coexistence of gambling and insurance preferences. Moreover, BPT not only integrates the mental accounting structure from Kahneman and Tversky's (1979), Tversky and Kahneman (1992) prospect theory, but also enables investors to consider their portfolio as a collection of subportfolios, each of which is optimal for a given mental account.

Our goal in this paper is to investigate the characteristics of the BPT optimal portfolio. Does the fact that the foundation of BPT is in sharp contradiction to that of MVT necessarily mean that the BPT optimal portfolio is different to that of the MV? In their seminal paper, Shefrin and Statman (2000) show that the BPT optimal portfolio is typically not MV efficient. However, this result has been questioned over recent years. A new stream of literature attempts to compare the asset allocation generated by BPT-like models with that generated by MVT (Alexander and Baptista, 2011; Baptista, 2012; Das et al., 2004; Levy and Levy, 2004; Levy et al., 2012). These studies provide evidence that some features of BPT and MVT almost make their asset allocations coincide. However, this does not provide sufficient evidence to conclude that investors can use BPT and MVT interchangeably. The

☆ This article was written in part when Tristan Roger was a visiting scholar at the Haas School of Business, University of California, Berkeley.
☆☆ The authors gratefully acknowledge the Editor Sushanta Mallick and the two anonymous referees for their valuable comments and suggestions.
E-mail address: m.pfiffelmann@unistra.fr (M. Pfiffelmann).
[1] See Ameur and Prigent (2013) for further details.

http://dx.doi.org/10.1016/j.econmod.2015.10.041
0264-9993/

weakness of all these studies lies in the assumption of normally distributed stock returns. This assumption has been shown to be unrealistic (Mandelbrot, 1963; Mantegna and Stanley, 1995). Moreover, none of these studies take the entire BPT framework into consideration.[2] It is therefore interesting to test if the asset allocations of these two models coincide when all these assumptions are relaxed. This is the first study to attempt an empirical determination of the BPT optimal portfolio. The characteristics of the BPT optimal portfolio are analyzed in terms of expected return, risk and skewness. BPT integrates the overweighting of small probabilities associated to extreme outcomes by investors. This behavior captures investor preference for positive skewness and lottery-like stocks (Bali et al., 2011; Kumar, 2009). The overweighting of small probabilities also reflects the inherent desire for safety in BPT investors (Lopes, 1987). These preferences will impact the expected return, risk and skewness of the optimal portfolio of investors.

Our analysis is comprised of three steps. In a first step, we investigate whether BPT and MVT lead to differences in individual portfolio choices. We compare the asset allocations generated by BPT and MVT without restrictions and with a sufficient number of stocks in the portfolio. To this end, we dismiss any assumption concerning the distribution of returns, take all aspects of BPT into account, and allow for short sales. Our approach is to empirically compute the asset allocations generated by BPT and by MVT and to locate the BPT optimal portfolio with respect to the MV frontier. To carry out our empirical study, we use the U.S. stock prices contained in the CRSP database to generate a universe of 100,000 possible asset allocations via bootstrap simulation. We solve the Shefrin and Statman (2000) optimization program for each date of our sample and determine the BPT optimal portfolio from the 100,000 portfolios. We establish that Shefrin and Statman's (2000) portfolio is MV efficient in over 70% of cases. BPT is developed on the foundations of Cumulative Prospect Theory (CPT), but does not incorporate all the features of this model. CPT is also characterized by the following criteria: (1) individuals maximize a value function based on gains and losses rather than final wealth, (2) gains and losses are defined with respect to a given reference point, and (3) the value function exhibits diminishing sensitivity and loss aversion. Our second analysis is to test whether the inclusion of these additional features modifies the MV efficiency of the optimal portfolio when computing the BPT portfolio. We transform the monetary outcomes of our portfolio sample via the CPT value function and determine a new optimal portfolio, namely the BPT_{CPT} optimal portfolio. We show that this new optimal portfolio leads to similar results in terms of MV efficiency. Despite different foundations, we show that MVT and BPT lead to similar portfolios in the MV space.

In a second step, we use an empirical approach to evaluate the three first moments of the portfolios selected by BPT investors. The BPT and BPT_{CPT} portfolios are always characterized by a high level of risk and high returns. These portfolios also display highly positive skewed returns for almost all of the period under study. We point out that the BPT optimal portfolio is absent during financial crises; none of the 100,000 portfolios in our sample meet the BPT safety-first constraint. This result is a consequence of the way BPT investors set their optimal portfolio. Investors initially satisfy the safety-first criteria at the cheapest price, deciding if they will invest and enter the market (concept of fear). BPT investors then use probability weighting, which captures the desire for positive skewness, to invest remaining wealth in an Arrow–Debreu security characterized by a high potential payoff (concept of potential). It follows that when the BPT optimal portfolio does exist, its returns are characterized by a positive skewness, a high level risk and high returns.

In a third step, we investigate whether efficient BPT and BPT_{CPT} portfolios would be chosen by typical Markowitz investors by examining the location of these portfolios on the MV efficient frontier. BPT and BPT_{CPT} portfolios are always characterized by a high level of risk and high returns, meaning that they always lie on the extreme upper right part of the frontier. We show that the risk aversion levels induced by BPT are incompatible with empirical observations. The risk aversion coefficient associated with the BPT optimal portfolio is up to 10 times lower than the degree of risk aversion shown by typical individual MV investors. As a consequence, typical Markowitz investors would typically avoid investing in such portfolios.

Our empirical analysis contributes to the literature in several ways. We find that the BPT (and BPT_{CPT}) optimal portfolio is efficient in most cases. We also show that even when they are efficient, BPT and BPT_{CPT} portfolios would not be chosen by typical investors as they are associated to a high level of risk and expected return. We underline that the BPT portfolio will be chosen by investors whose behavior is radically different from that of MV investors. The BPT and BPT_{CPT} portfolios are chosen by investors who are attracted by positively skewed returns and exhibit risk-seeking behavior when potentially high gains are reachable. However, these portfolios will not be chosen by investors when the potential losses are too great.

The paper is structured as follows: Section 2 reviews the main characteristics of Shefrin and Statman's (2000) model and the related literature. Section 3 presents the data and describes our methodology. Section 4 provides the results of this empirical study. Section 5 discusses the features of the BPT and BPT_{CPT} portfolios. Section 6 presents the robustness tests. Section 7 concludes with a summary of our findings.

2. The model

The Behavioral Portfolio Theory (BPT) developed by Shefrin and Statman (2000) is based on Roy's (1952) concept of the safety-first approach. This approach implies that the investor's portfolio risk is not measured by the variance, but rather by the probability of ruin.[3] Ruin is considered to occur when the investor's final wealth W falls below the subsistence level. The idea underlying Roy's (1952) concept is that investors aim to minimize the probability of ruin. Telser (1955) extends Roy's (1952) concept by introducing the idea of an acceptable level for the probability of ruin. A portfolio is considered safe when the probability of ruin does not exceed a given level α. In Telser's (1955) model, investors are concerned about both the expected return of the portfolio and the probability of failure to reach the given subsistence level s.[4] It follows that investors aim to maximize their expected return while keeping the probability of ruin below a given α level. Formally, the model is shown as

$$\max E(R)\,\text{u.c.}\, P(W < s) < \alpha, \tag{1}$$

where R is the portfolio return, s is the subsistence level, α is the acceptable probability of ruin, and W is the final wealth distribution. BPT is based on Roy's (1952) safety-first approach, but also integrates some features of behavioral economics and finance. It combines Lopes' (1987) Security Potential and Aspiration Theory (commonly denoted SP/A) with the mental accounting structure from Kahneman and Tversky's (1979, 1992) prospect theory. In SP/A theory, the investor's choice is driven by three factors, namely security (S), potential (P) and aspiration (A). The security factor and the potential factor relate to two emotional drivers, fear and hope. On the one hand, investors are driven by fear and wish to secure their wealth. On the other hand, they are willing to take risks to increase the potential gains. These two

[2] To date, the only comparative analysis carried out without any prior assumption about return distribution is the empirical study realized by Hens and Mayor (2014). However, like the studies mentioned above, Hens and Mayor (2014) do not take the entire BPT framework into account.

[3] The concept of ruin corresponds to the failure to reach a given subsistence level.

[4] These investors therefore act as MVT investors, since they are attentive to the expected return of the portfolio and its risk. However, contrary to MVT, the risk of the portfolio is not defined by the variance of the portfolio returns but rather by the downside risk.

emotional drivers are expressed in the model through a modified probability distribution of outcomes. Fear (or hope) operates through an overweighting of the small probabilities associated with the worst (or best) outcomes. Thus, investors compute their expected wealth by applying an inverse S-Shape weighting function to the decumulative probability distribution of outcomes, and by substituting $E_\pi(W)$ for $E(W)$. Lopes' (1987) concept of aspiration generalizes the concept of subsistence level as described above. Investors aim to reach a specific level for their final wealth, called the aspiration level or target value. The risk of the portfolio is therefore the probability of ending below this target value A. BPT incorporates the mental accounting structure of Kahneman and Tversky's (1979) prospect theory. Investors have distinct mental accounts (e.g. education, retirement, bequest…) with different levels of aspiration. They do not consider their portfolio as a whole but rather as a collection of mental accounting subportfolios with distinct aspiration levels. In BPT, investors maximize their expected wealth (calculated with decision weights) subject to the constraint of the probability of failure reaching a threshold level A that remains below a given α level. This optimization program is as follows

$$\max E_\pi(W) \text{u.c.}\, p(W<A)<\alpha, \tag{2}$$

where A is the aspiration level, α is the maximum probability of ruin, W is the final wealth distribution and, π is a transformation function of probabilities. The literature proposes several ways to transform probabilities. This study uses the specification proposed by Tversky and Kahneman (1992) in Cumulative Prospect Theory. The model is detailed in 8. In theory, the payoff of the BPT portfolio can be seen as the payoff of a portfolio combining bonds and a lottery ticket. This particular payoff results from the decision process of BPT investors; they do not allocate their wealth simply by solving a mean-variance optimization problem. Their portfolio can be viewed as a pyramid of assets, where the riskless instruments are at the bottom and the riskier assets are at the top. BPT investors proceed in two steps to set their portfolios. First, they satisfy the safety-first criteria at the cheapest price (concept of security), then they invest the remaining wealth in an Arrow Debreu security characterized by a high potential payoff (concept of potential). In their seminal paper, Shefrin and Statman (2000) show that the BPT optimal portfolio is different from the Markowitz optimal portfolio. However, this result has been questioned over recent years. Several recent studies attempt to compare the asset allocation generated by BPT-like models with that generated by MVT. Levy and Levy (2004) show that while prospect theory findings are in contradiction with the foundations of MVT, the prospect theory and MV efficient sets can coincide. Das et al. (2004) integrate appealing features of MVT and BPT into a new mental accounting (MA) framework. The authors assume that a rational investor divides her wealth among several mental accounts and seeks to reach a threshold in each mental account. They demonstrate that the MA optimal portfolio always lies on the MV efficient frontier. Like Das et al. (2004), Alexander and Baptista (2011) develop a model where the investor divides her wealth among accounts, but they also assume that she delegates the task of allocating wealth among assets to portfolio managers. They show that portfolio delegation with mental accounting leads an investor to select optimal portfolios within accounts that generally lie far from the MV frontier. Baptista (2012) extends Das et al. (2004) model by assuming that investors face background risk in addition to portfolio risk. He shows that optimal portfolios lie far from the MV frontier under fairly general conditions. Jiang et al. (2013) analyze international portfolio selection with exchange rate risk based on BPT. They show that the optimal BPT portfolio with exchange rate risk is typically not mean-variance efficient from the perspective of local investors unless certain conditions are satisfied. Levy et al. (2012) show that the Security Market Line Theorem of the Capital Asset Pricing Model (CAPM) is intact in the Cumulative Prospect Theory (CPT) framework (Tversky and Kahneman, 1992). With regard to these studies, some features of BPT and MVT almost make the asset allocations coincide. Yet these results are not sufficient evidence to conclude that practitioners can use BPT and MVT interchangeably. The weakness of all these studies is that they are based on strong assumptions and do not take the whole BPT framework into account. For instance, Levy and Levy (2004) and Levy et al. (2012) compare MVT with prospect theory and with Cumulative Prospect Theory. In these studies, the authors integrate behavioral aspects of BPT but do not take into consideration the safety-first criteria, which is a key attribute of BPT. In contrast, Das et al. (2004), Alexander and Baptista (2011), Baptista (2012) and Jiang et al. (2013) take the mental accounting framework and the safety first criteria of BPT into account, but do not integrate the behavioral features of BPT, whereby investors transform objective probabilities into decision weights. Moreover, all these studies are based on the same strong assumption of normally distributed stock returns.[5] This assumption has been shown to be unrealistic (Mandelbrot, 1963; Mantegna and Stanley, 1995). To the best of our knowledge, the only existing comparative analysis carried out without any prior assumption about return distribution is the empirical study by Hens and Mayor (2014). These authors show that the asset allocation derived for CPT differs substantially from the MV analysis when asset returns are not normally distributed. Their empirical study has the advantage of discarding the assumption of normally distributed returns. However, the validity of their results is limited by the fact that their data set is based on an empirical distribution carried out with only 8 assets and 15 realizations. This small number of stocks in the portfolios does not permit a good level of diversification. Moreover, like Levy and Levy (2004) and Levy et al. (2012), Hens and Mayor (2014) do not take the entire BPT framework into consideration. In their study, they integrate the behavioral aspects of BPT but do not take the safety-first criteria into consideration, although it is a key attribute of BPT.

3. Data and methodology

3.1. Data

The primary dataset used in this article is comprised of the daily stock prices of 1,452 U.S. stocks from the CRSP database, with a complete price history for the 1995–2011 period. We use stock prices and dividends paid by the firms to compute the monthly stock returns[6] on a daily frequency (rolling windows). The monthly return $R_{t,i}$ on stock i for day t is calculated as

$$R_{t,i} = \log(P_{t,i} + D_{t,i}) - \log(P_{t-20,i}), \tag{3}$$

where $P_{t,i}$ is the price of stock i for day t and $D_{i,t}$ is the dividend paid by the firm.

3.2. The bootstrap method

We consider the matrix R, which contains the 4262 monthly stock returns over the 1995–2011 period. R is given by

$$R = \begin{bmatrix} R_{1,1} & R_{1,2} & \dots & R_{1,1492} \\ R_{2,1} & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ R_{4262,1} & \dots & \dots & R_{4262,1492} \end{bmatrix}, \tag{4}$$

where $R_{1,1}$ is the monthly return on the first stock over the period starting on the 3rd of January 1995 and ending on the 31st of January 1995, $R_{2,2}$ is the monthly return of the second stock over the period starting on the 4th of January 1995 and ending on the 1st of February 1995, and so on.

[5] Das and Statman (2013) underline that the optimization program of the MA model (Das et al., 2004) is a special case of the MV model under normality.

[6] Obtaining 4262 monthly returns.

BPT investors determine their optimal portfolio by maximizing an objective function based on subjective expected portfolio returns. To determine the BPT optimal portfolio, we therefore assume a single period economy and generate a series of expected returns from historical returns. Our aim is to use the series of historical ex-post returns as an input to compute portfolio expected returns. Our analysis relies on a rolling sample approach. Specifically, for each date t (from $t = 251$ to $t = 4262$), we generate possible scenarios (states of nature). To this end, we use the bootstrap historical simulation method (Hull and White, 1998). Our approach is the following. In each of the 4012 simulations (from $t = 251$ to $t = 4262$), we first randomly select 80 stocks[7] among the 1492 making up our sample.[8] We then select the 250 monthly returns of these 80 stocks prior to date t. We obtain a matrix R^*, defined as

$$R^* = \begin{bmatrix} R^*_{t-250,1} & R^*_{t-250,2} & \cdots & R^*_{t-250,80} \\ R^*_{t-249,1} & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ R^*_{t-1,1} & \cdots & \cdots & R^*_{t-1,80} \end{bmatrix}. \tag{5}$$

To model the first state of nature, i.e., the return obtained at the end of our single period, we randomly select a row of the matrix R^*. We repeat this process 1000 times in order to obtain the 1000 states of nature for our 80 stocks. Note that by randomly selecting a row of the matrix R^*, we do not alter the structure of correlations between the different stocks. The matrix θ containing the 1000 states of nature for the 80 stocks at date t is as follows

$$\theta = \begin{bmatrix} \theta_{1,1} & \theta_{1,2} & \cdots & \theta_{1,80} \\ \theta_{2,1} & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \theta_{1000,1} & \cdots & \cdots & \theta_{1000,80} \end{bmatrix}, \tag{6}$$

where $\theta_{i,j}$ is the monthly return of stock j if state of nature i occurs.

We repeat this process 4012 times (from $t = 251$ to $t = 4262$) to obtain a significant number of different BPT optimal portfolios.

3.3. Generation of portfolios

In practice, for each date t, there is an infinite number of different portfolios that the investor can choose to hold. However, for our study, we build a set of 100,000 portfolios. The investor will then choose one portfolio among the 100,000 possible choices. We also limit to 80 the maximum number of stocks that the investor can hold. This large number of stocks in the portfolio ensures a good level of portfolio diversification. The methodology presented below was used to generate the sample of 100,000 portfolios and provide the best possible approximation of the choice faced by the investor in reality. We consider two different situations. In the first situation, the investor cannot short stocks. In the second situation, short sales are possible.

3.3.1. Portfolios without short sales

In order to obtain a good diversity between the different portfolios, we need to generate portfolios with different numbers of stocks and different weight distributions. Our approach is the following. We consider a portfolio of up to n stocks. We assume that the weight associated to a given stock is equal to k/n with $k = 0, 1, \ldots, n$. It follows that the most diversified portfolio is the $1/n$ portfolio, while the least diversified portfolio is the one where the investor invests all her wealth in just one stock. Given the assumption that the weight associated to the different stocks is equal to k/n, we can model all the different possible portfolio compositions by considering all the possible integer decompositions (without ranking) of the number n. For example, for $n = 4$ there are 5 possible integer decompositions:

$$I_1 = \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad I_2 = \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad I_3 = \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix} \quad I_4 = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix} \quad I_5 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}. \tag{7}$$

If we want to randomly select a portfolio of up to four stocks, we select one of the five possible combinations (see above) and then randomly reorder the vector of weights. In other words, we first select the structure of weights, then randomly attribute these different weights to the different stocks.[9] For $n = 80$, the total number of integer decompositions is equal to 15,796,476 (see, Rademacher, 1937).[10] We then randomly select 100,000 decompositions among the 15,796,476 possible integer decompositions, randomly reorder these vectors and transform them into weights by dividing each element by $n = 80$.

The same 100,000 weight vectors are used for each simulation (note that the 80 stocks are different for each simulation). The average number of stocks in the portfolios is equal to 18.18 (median of 18), with a maximum of 70 stocks and a minimum of 3. We calculate the diversification of each portfolio by means of the Herfindahl index.[11] The mean Herfindahl index is equal to 0.1271 (median of 0.1163). The most (respectively least) diversified portfolio has a Herfindahl index of 0.0178 (respectively 0.7037).

3.3.2. Portfolios with short sales

We now consider a scenario where our investor is allowed to short stocks. Our methodology to compute a portfolio when short sales are allowed is as follows: We randomly select three portfolios A, B and C following the above methodology. A portfolio with negative weights is simply obtained by considering the linear combination $A + B - C$. The sum of the weights of $B - C$ is equal to 0. It follows that the sum of weights of the combination $A + B - C$ is equal to one. Let us illustrate this combination by considering the following portfolios A, B and C:

$$A = \begin{bmatrix} 0.25 \\ 0.25 \\ 0.5 \\ 0 \end{bmatrix} \quad B = \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{bmatrix} \quad C = \begin{bmatrix} 0 \\ 0.25 \\ 0 \\ 0.75 \end{bmatrix} \quad \Rightarrow \quad A + B - C = \begin{bmatrix} 0.5 \\ 0.25 \\ 0.75 \\ -0.5 \end{bmatrix}. \tag{8}$$

The result of this linear combination is a portfolio containing both positive and negative weights.

When allowing for short sales, the algorithm generates portfolios that contain more stocks than they do when short sales are constrained. The average number of stocks in the portfolios is equal to 42.87 (median of 43) with a maximum of 73 stocks and a minimum of 19. On average, the portfolios contain 29 long positions (mean of 29.91 and median of 28) and 14 short positions (mean of 13.96 and median of 13). In the portfolios, the average ratio of long positions to short positions is equal to 2.36 (median of 2.14) with a minimum ratio of 0.33 and a maximum ratio of 22.

[7] In Section 6, we present robustness tests where the 80 stocks are not randomly selected.

[8] Portfolios of up to 80 stocks provide sufficient diversification. Additionally, computation for portfolios containing over 80 stocks becomes extremely long or even impossible.

[9] Our baseline approach implies that each of the 1452 stocks has the same probability of being in the investor's portfolio. In a robustness check presented at the end of the article, we introduce a tilt toward large capitalization stocks to reflect the reality that not every company has the same frequency of being in the investor's portfolio.

[10] The algorithm used to generate the integer decompositions is available upon request.

[11] The Herfindahl index of diversification is equal to

$$HI = \sum_{i=1}^{n} (\omega_i)^2,$$

where n is the number of stocks in the portfolio and ω_i is the weight of stock i.

3.4. The BPT optimal portfolio and the efficient frontier

In this section, our aim is to empirically determine the BPT optimal portfolio and to locate it in the MV space. We make an empirical estimation of the efficient frontier using the 100,000 generated portfolios. The approach is as follows: For each portfolio, we check if there is another portfolio with a higher expected return and a lower variance. A portfolio is considered to be located on the efficient frontier if no other portfolio is found in the sample with both a higher expected return and a lower variance. We call the set of portfolios that are located on the efficient frontier S^{ef}.

The BPT optimal portfolio satisfies

$$\max E_{\pi}(W) u.c. p(W<A)<\alpha, \tag{9}$$

where W is the final wealth distribution of the investor, A is the aspiration level and α the acceptable probability of ruin.

We first determine the portfolios that satisfy the safety first constraint. A portfolio satisfies this constraint if its value at the end of the period is at least equal to aspiration level A in $1-\alpha\%$ of the states of nature. We denote S^* as the set of portfolios satisfying the safety-first constraint. By construction, $S^* \subset S$, S being the set of 100,000 generated portfolios. For illustrative purposes, Fig. 1 plots set S, composed of the 100,000 generated portfolios (black crosses), and set S^*, composed of the portfolios satisfying the safety-first constraint (gray stars).

The BPT optimal portfolio is the S^* portfolio that maximizes $E_{\pi}(W)$. In order to compute the expected return with subjective decision weights, we need to transform the objective probabilities (denoted p_i) into decision weights via a weighting function. BPT is a rank-dependent model, as defined by Quiggin (1982). This means that the weighting function w is not applied to individual objective probabilities, but rather to the cumulative or decumulative distribution function of the prospect. The methodology for transforming the probabilities is as follows: Firstly, the vector of states of nature θ for each portfolio were ranked from the worst outcome to the best outcome (see Eq. (6)). For portfolio i, the vector of states of nature ranked from the worst outcome to the best outcome is stated as

$$y_i = \begin{pmatrix} y_{i,1} \\ \cdots \\ \cdots \\ y_{i,1000} \end{pmatrix}, \tag{10}$$

where $y_{i,k}$ is the value of portfolio i if the kth state of nature is realized.

Secondly, we transform the objective probabilities into decision weights. We accomplish this transformation by applying the weighting function proposed by Tversky and Kahneman (1992) in CPT. The bootstrap simulation assumes that each state of nature is equally likely. Thus, the vector p of objective probabilities is written as

$$p = \begin{pmatrix} p_1 = \dfrac{1}{1000} \\ \cdots \\ \cdots \\ p_{1000} = \dfrac{1}{1000} \end{pmatrix}. \tag{11}$$

We define the vector of decision weights as

$$\pi = \begin{pmatrix} \pi_1 \\ \vdots \\ \pi_k \\ \vdots \\ \pi_{1000} \end{pmatrix} = \begin{pmatrix} \pi_1 = w\left(\sum_{j=1}^{1000} p_j\right) - w\left(\sum_{j=2}^{1000} p_j\right) \\ \vdots \\ \pi_k = w\left(\sum_{j=i}^{1000} p_j\right) - w\left(\sum_{j=k+1}^{1000} p_j\right) \\ \vdots \\ \pi_{1000} = w(p_{1000}) \end{pmatrix}, \tag{12}$$

where w is the weighting function[12] proposed by Tversky and Kahneman (1992). Consider F the cumulative distribution function of y. We notice that $w(pi)$ is applied to the decumulative distribution function of y.

$$w\left(\sum_{j=1}^{n} p_j\right) = w\left(1 - F\left(y_{j-1}\right)\right) \tag{13}$$

This methodology permits to have different levels of decision weights even though the objective probabilities are equiprobable.

The BPT optimal portfolio is the S^* portfolio that maximizes the inner product $E_{\pi}(W) = y'\pi$. The final step is to check whether this portfolio is part of the S^{ef} set (the set that contains the portfolios located on the efficient frontier). We repeat this process 4012 times (for each date t) in order to obtain a significant number of optimal portfolios.

3.5. Transformation of the monetary outcomes: the BPT_{CPT} portfolio

In BPT, Shefrin and Statman (2000) consider an investor who transforms probabilities into decision weights. But behavioral studies such as CPT show that the distortion of objective probabilities is not the only irrational feature of investors.[13] Investors also make decisions based on change of wealth rather than on total wealth, and can exhibit risk-seeking behavior when faced with losses. Therefore, individuals determine the subjective value of each monetary outcome via a value function. Our aim is to check if the same results occur for a CPT investor who subjectively transforms monetary outcomes and objectives probabilities. The BPT_{CPT} optimal portfolio is the portfolio that satisfies

$$\max E_{\overline{\pi}}[v(W)] u.c. p(W<A)<\alpha, \tag{14}$$

where v is the CPT value function that transforms the monetary outcomes into utility.[14]

The function v is defined as relative to a reference point κ that distinguishes between gains and losses. In this study, we assimilate the reference point κ to the long-term risk-free rate (i.e., 10-year U.S. Treasury Bond).[15] A stock return greater than the long-term risk-free rate is considered a gain, whereas a stock return below long-term risk-free rate is considered a loss.

The methodology is identical to that used in the previous section. The difference is that the BPT_{CPT} optimal portfolio is the S^* portfolio that maximizes the inner product $E_{\pi}[v(W)] = v'\overline{\pi}$. For portfolio i, the vector of modified (ranked) outcomes is defined as

$$v_i = \begin{pmatrix} v\left(y_{i,1}\right) \\ \cdots \\ \cdots \\ v\left(y_{i,n}\right) \end{pmatrix}. \tag{15}$$

As gains and losses are treated differently in the value function, an identical approach is necessary for the weighting function. We define

[12] The weighting function w is

$$w(p) = \frac{p^{\gamma}}{\left[p^{\gamma} + (1-p)^{\gamma}\right]^{1/\gamma}},$$

with $\gamma = 0.61$.

[13] All the features of CPT are detailed in Appendix A.

[14] The CPT value function is defined by

$$v(y) = \begin{Bmatrix} (y-\kappa)^{0.88} & \text{if } y \geq \kappa \\ -2.25(-(y-\kappa))^{0.88} & \text{if } y < \kappa \end{Bmatrix}.$$

[15] In a robustness check presented at the end of the article, we assimilate the reference point to the average return of the S&P 500 over the previous three years.

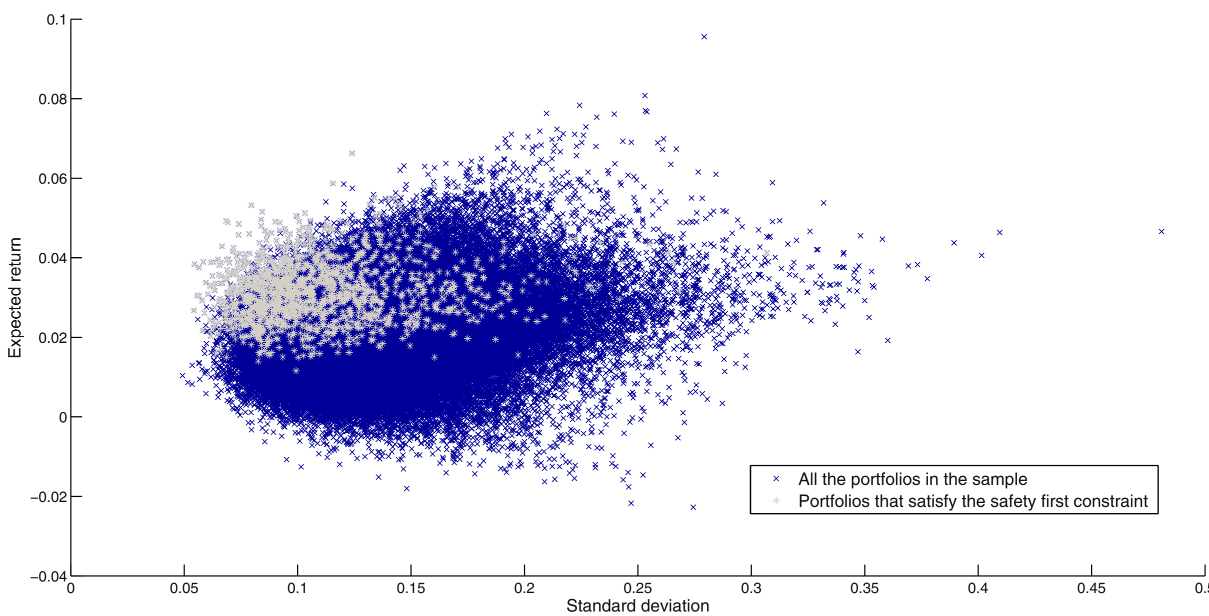

Fig. 1. Illustration: portfolios satisfying the safety first constraint.

the modified vector of decision weights $\overline{\pi}$ as

$$\overline{\pi} = \begin{pmatrix} \overline{\pi}_1^- \\ \cdots \\ \overline{\pi}_l^- \\ \cdots \\ \overline{\pi}_k^+ \\ \cdots \\ \overline{\pi}_n^+ \end{pmatrix} = \left(\begin{array}{c} \overline{\pi}_1^- = w^-(p_1) \\ \cdots \\ \overline{\pi}_l^- = w^-\left(\sum_{j=1}^{l} p_j\right) - w^-\left(\sum_{j=1}^{l-1} p_j\right) \\ \cdots \\ \overline{\pi}_k^+ = w^+\left(\sum_{j=k}^{n} p_j\right) - w^+\left(\sum_{j=k+1}^{n} p_j\right) \\ \cdots \\ \overline{\pi}_n^+ = w^+(p_n) \end{array} \right) \tag{16}$$

where w is the weighting function[16] proposed by Tversky and Kahneman (1992).

The optimal portfolio is the portfolio that maximizes the inner product $E_\pi[v(W)] = v'\overline{\pi}$. As in Section 3.4, we repeat this simulation 4,012 times in order to obtain a significant number of optimal portfolios.

4. Empirical analysis

Two samples of 100,000 portfolios are considered here, namely one without short sales and one where short sales are allowed. Each investor sets a safety-first constraint by specifying the return on a portfolio which should not fall below level A with more than α probability. Because the safety-first constraint is not necessarily the same for each investor or each mental account, we consider several configurations for α and A. The aspiration level A is given by the initial wealth, capitalized at a rate r. We consider six different specifications for r: (1) the short-term risk free rate (i.e., 3-month Treasury Bill); (2) the long-term risk-free rate (i.e., 10-year U.S. Treasury Bond); (3) the average S&P 500 return over the previous three years; (4) an annualized rate of 1%; (5) an annualized rate of 5%; and, (6) an annualized rate of 10%. The different values for α are 0.1, 0.2 and 0.3. For instance, values set as $r = 1\,\%$ and $\alpha = 0.1$ indicate the investor's desire for the probability of failing to reach $A = W_0 e^{1+r}$ to exceed $\alpha = 0.1$.[17] We have a total of 18 different specifications.

4.1. Non-Gaussian portfolios returns

For each simulation, we run both a Jarque–Bera test and a Kolmogorov–Smirnov test to check whether the portfolio returns are Gaussian. The Jarque–Bera test indicates that, for each simulation, about 90,000 portfolios out of 100,000 present non-Gaussian returns (at a 5% significance level). When we use the Kolmogorov–Smirnov test, all the portfolios have non-Gaussian returns.

4.2. Safety first constraint

The number of portfolios (among the 100,000) that meet the safety-first constraint are checked or each simulation. Fig. 2 indicates the number of simulations with at least one portfolio meeting the safety-first constraint for different parameterizations of α and different aspiration levels. The number of simulations with at least one portfolio meeting the safety-first constraint increases with α and decreases with the level of the aspiration level. With α equal to 0.1 and the aspiration level set as a short-term risk-free rate, at least one portfolio meets the constraint in 26% of the cases. When choosing α equal to 0.3 and the same aspiration level, this proportion increases to 92.3%. This result seems quite natural, since the expectation of investors decreases with α. When α decreases, the investor wants to secure more states of nature. Conversely, if α increases, the investor wants to secure fewer states of nature. It is therefore more likely that a portfolio will satisfy the safety-first constraint when α is high. For our empirical analysis, we only consider simulations in which at least one portfolio meets the safety-first constraint.

[16] The weighting function w is written as

$$w^+(p) = \frac{p^{\gamma+}}{\left[p^{\gamma+} + (1-p)^{\gamma+}\right]^{1/\gamma+}}$$
$$w^-(p) = \frac{p^{\gamma-}}{\left[p^{\gamma-} + (1-p)^{\gamma-}\right]^{1/\gamma-}}$$

where $\gamma^+ = 0.61$ and $\gamma^- = 0.69$.

[17] For simplicity's sake, in the rest of the paper we will write $A = 1\,\%$ when we set the rate r as equal to 1%.

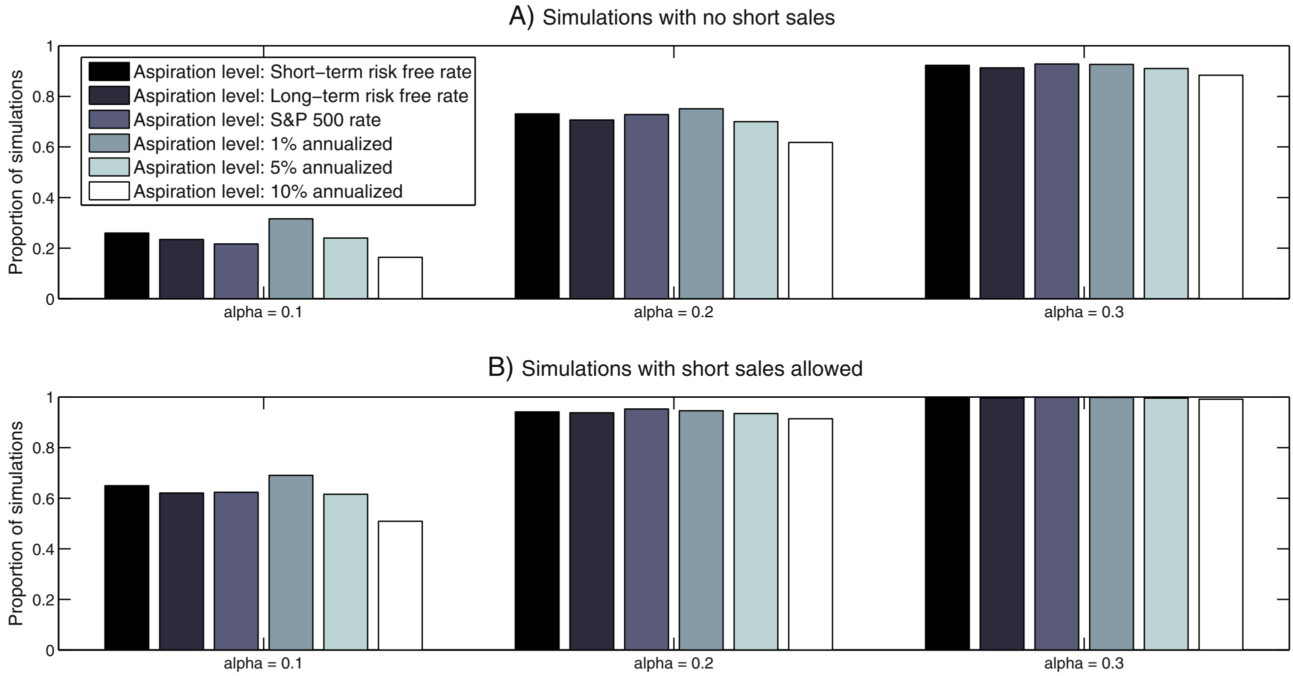

Fig. 2. Proportion of simulations with at least one portfolio meeting the safety first constraint.

4.3. Optimal portfolios and the efficient frontier

Table 1 indicates the proportion of simulations for which the BPT optimal portfolio is located on the efficient frontier. We observe that the BPT optimal portfolio is located on the efficient frontier in approximately three out of four simulations. This proportion is relatively robust to changes in α or in the aspiration level. The introduction of short sales does not appear to modify the results.

Table 2 indicates the proportion of simulations for which the BPT_{CPT} optimal portfolio is located on the efficient frontier. The results are similar to those obtained when studying the BPT optimal portfolio. The BPT_{CPT} optimal portfolio is located on the efficient frontier in a little over three out of four simulations. This proportion is relatively robust to changes in α or in the aspiration level. Here again, the introduction of short selling does not modify the results.

5. Characteristics of the BPT portfolios

In this section, we investigate the location of the BPT and BPT_{CPT} optimal portfolios in the MV space. First, we locate the BPT optimal portfolio relative to the BPT_{CPT} optimal portfolio. We examine whether these portfolios coincide or whether one is always riskier than the other. We then look at the expected return, standard deviation and skewness of the BPT and BPT_{CPT} optimal portfolios. Finally, we investigate the location of the BPT and BPT_{CPT} optimal portfolios on the MV efficient frontier.

Table 1
Proportion of simulations for which the BPT optimal portfolio is MV efficient.

Aspiration level	r_{ST}	r_{LT}	$r_{S\&P}$	$r_{1\%}$	$r_{5\%}$	$r_{10\%}$
Panel A: Short sales forbidden						
$\alpha = 0.1$	0.7284	0.7316	0.7215	0.7380	0.7287	0.7321
$\alpha = 0.2$	0.7369	0.7245	0.7255	0.7272	0.7291	0.7357
$\alpha = 0.3$	0.6975	0.7051	0.7037	0.6933	0.7065	0.7171
Panel B: Short sales allowed						
$\alpha = 0.1$	0.7694	0.7659	0.7929	0.7627	0.7688	0.8070
$\alpha = 0.2$	0.7260	0.7291	0.7319	0.7249	0.7249	0.7491
$\alpha = 0.3$	0.6722	0.6744	0.6745	0.6654	0.6732	0.6845

This table provides the proportion of simulations for which the BPT optimal portfolio is MV efficient. The probability of failure (α) to reach the aspiration level takes the value 0.1, 0.2 and 0.3. The different values for the aspiration level are: (1) the long-term risk-free rate (r_{ST}); (2) the long-term risk-free rate (r_{LT}); (3) the S&P 500 return over the past 3 years ($r_{S\&P}$); (4) an annualized rate of 1% ($r_{1\%}$); (5) an annualized rate of 5% ($r_{5\%}$); and, (6) an annualized rate of 10% ($r_{10\%}$). The total number of simulations is 4,012. For each simulation, the BPT optimal portfolio is selected from a sample of 100,000 portfolios.

5.1. Location of BPT portfolios relative to BPT_{CPT} portfolios

Table 3 displays the proportion of simulations for which BPT and BPT_{CPT} optimal portfolios coincide. For instance, when short sales are allowed, the aspiration level corresponds to the long-term risk-free rate and $\alpha = 0.2$, BPT and BPT_{CPT} optimal portfolios coincide in 67% of cases. In the remaining 33%, the BPT optimal portfolio has a higher variance than the BPT_{CPT} optimal portfolio (in more than 99% of cases).[18] This difference in variances is significant at the 1% level.[19] The BPT optimal portfolio appears to be riskier than the BPT_{CPT} optimal portfolio. An explanation for this result is that BPT_{CPT} investors transform monetary outcomes into utility via a value function v. This function v is defined to integrate the behavioral observations of Tversky and Kahneman (1992). First, v is concave over gains and convex over losses. That means that BPT_{CPT} investors are characterized by risk averse behavior for most gains (gains associated with moderate and high probabilities). Second, v is steeper for losses than for gains to account for the fact that investors have an asymmetric perception of gains and losses. Experimental evidence (Erev et al., 2008; Tversky and Kahneman, 1991) shows that individuals are loss averse; the loss of a given amount of money creates distress that has a greater effect than the satisfaction generated by a gain of the same amount. Losses are weighted about twice as much as gains. Thus, BPT_{CPT} investors are characterized by risk averse behavior for most gains, and a strong loss aversion. It seems natural, therefore, that these agents select less risky portfolios than BPT investors.

[18] This result is robust for all the other specifications. On average, the BPT optimal portfolio has a higher variance than the BPT_{CPT} optimal portfolio in more than 98% of cases.

[19] We run a Jarque–Bera test and a Kolmogorov–Smirnov test to check that the series of variances are normally distributed, then use a Student t-test to compare the mean variances of the BPT and BPT_{CPT} optimal portfolios.

Table 2
Proportion of simulations for which the BPT_{CPT} optimal portfolio is MV efficient.

Aspiration level	r_{ST}	r_{LT}	$r_{S\&P}$	$r_{1\%}$	$r_{5\%}$	$r_{10\%}$
Panel A: Short sales forbidden						
$\alpha = 0.1$	0.7610	0.7572	0.7457	0.7632	0.7599	0.7504
$\alpha = 0.2$	0.7731	0.7647	0.7745	0.7690	0.7672	0.7647
$\alpha = 0.3$	0.7726	0.7799	0.7709	0.7789	0.7807	0.7800
Panel B: Short sales allowed						
$\alpha = 0.1$	0.7939	0.7900	0.8145	0.7920	0.7927	82.61
$\alpha = 0.2$	0.7818	0.7828	0.7837	0.7834	0.7822	0.7906
$\alpha = 0.3$	0.7603	0.7617	0.7643	0.7586	0.7905	0.7642

This table provides the proportion of simulations for which the BPT_{CPT} optimal portfolio is MV efficient. The probability of failure (α) to reach the aspiration level takes the value 0.1, 0.2 and 0.3. The different values for the aspiration level are: (1) the long-term risk-free rate (r_{ST}); (2) the long-term risk-free rate (r_{LT}); (3) the S&P 500 return over the past 3 years ($r_{S\ \&P}$); (4) an annualized rate of 1% ($r_{1\ \%}$); (5) an annualized rate of 5% ($r_{5\ \%}$); and, (6) an annualized rate of 10% ($r_{10\ \%}$). The total number of simulations is 4,012. For each simulation, the BPT_{CPT} optimal portfolio is selected from a sample of 100,000 portfolios.

Table 3
Proportion of simulations for which the BPT optimal portfolio is the same portfolio as the BPT_{CPT} optimal portfolio.

Aspiration level	r_{ST}	r_{LT}	$r_{S\&P}$	$r_{1\%}$	$r_{5\%}$	$r_{10\%}$
Panel A: Short sales forbidden						
$\alpha = 0.1$	0.8916	0.8924	0.8953	0.8769	0.8898	0.9178
$\alpha = 0.2$	0.6987	0.7108	0.7190	0.6791	0.7063	0.7413
$\alpha = 0.3$	0.4999	0.5035	0.5111	0.4886	0.5079	0.5372
Panel B: Short sales allowed						
$\alpha = 0.1$	0.9025	0.9012	0.8992	0.8902	0.9065	0.9128
$\alpha = 0.2$	0.6725	0.6831	0.6831	0.6661	0.6796	0.7028
$\alpha = 0.3$	0.5328	0.5394	0.5321	0.5311	0.5408	0.5528

This table provides the proportion of simulations for which the BPT optimal portfolio is identical to the BPT_{CPT} optimal portfolio. The probability of failure (α) to reach the aspiration level takes the value 0.1, 0.2 and 0.3. The different values for the aspiration level are: (1) the long-term risk-free rate (r_{ST}); (2) the long-term risk-free rate (r_{LT}); (3) the S&P 500 return over the past 3 years ($r_{S\ \&P}$) (4) an annualized rate of 1% ($r_{1\ \%}$); (5) an annualized rate of 5% ($r_{5\ \%}$); and, (6) an annualized rate of 10% ($r_{10\ \%}$). The total number of simulations is 4,012. For each simulation, the BPT and the BPT_{CPT} optimal portfolios are selected from a sample of 100,000 portfolios.

In Table 3 we observe that the percentage of simulations for which BPT and BPT_{CPT} optimal portfolios coincide decreases with α levels. When the aspiration level corresponds to the long-term risk-free rate, α equals 0.3 and short-sales are allowed, BPT and BPT_{CPT} portfolios coincide in 54% of the simulations. This result is not surprising as fewer portfolios meet the safety-first constraint when the α level is low.

5.2. Location of the portfolios on the efficient frontier

BPT and BPT_{CPT} portfolios consistently exhibit a high expected return and a high level of risk in all 18 specifications. On average, the expected return of the BPT portfolio is 10 times higher than the return of the S&P 500 when short sales are allowed, and 5 times higher without short sales.[20] A significant increase in expected return and the risk of the BPT portfolio is observed when there are no short selling constraints. This difference is due to the way BPT investors select their portfolio. They first secure wealth with respect to their aspiration level. Next, they bet in few states of nature in order to meet their expectations of growing rich in a sizable way. In this second phase, BPT investors are willing to take more risks in the hope of winning a significant amount of money. The possibility of short-selling enhances this risk-taking behavior.

In Fig. 3, we plot the expected return and standard deviation of the BPT optimal portfolio for each date t between 1996 and 2011.[21] On this graph, we also represent the expected returns and standard deviations of the maximum return portfolio (i.e., the portfolio located on the efficient frontier which presents the highest return) and of the minimum variance portfolio. The expected return and risk associated with the BPT optimal portfolio are close to those of the maximum return portfolio. This observation is consistent with the concept of hope and potential inherent in Shefrin and Statman's (2000) model. Investors aim to grow rich in a sizable way and are willing to take on risk to increase potential gains (Lopes, 1987). The skewness of the BPT optimal portfolio confirms this point. Fig. 4 plots the skewness of the BPT optimal portfolio and the average skewness of the portfolios located on the mean variance frontier. The BPT portfolio displays positively skewed returns. The concept of hope inherent to BPT investors leads them to gamble and to prefer positively skewed portfolios in the hope of obtaining a high return. Such investors will prefer lottery-type stocks with highly positive skewness (Bali et al., 2011; Kumar, 2009). Figs. 3 and 4 also show the absence of a BPT optimal portfolio for several dates of our sample (2003, 2009). This absence coincides with periods of financial crises where expected returns are relatively low. During such periods, BPT investors decide against invest in stocks. This behavior results from the concept of fear and security that characterizes BPT investors. BPT investors wish to secure their wealth before gambling and investing in skewed assets. If no potential portfolio can satisfy their safety-first constraint, they do not take on risk and exit the market.

5.3. BPT portfolio and mean variance investors

The previous sections show that most BPT and BPT_{CPT} optimal portfolios are MV efficient. However, while being efficient, these portfolios are characterized by a very high level of risk. They are therefore always located on the upper right of the efficient frontier. Fig. 5 provides an illustration of the empirical efficient frontier and of the BPT optimal portfolio location. When BPT and BPT_{CPT} optimal portfolios are MV efficient, they coincide with portfolios that could theoretically be chosen by Markowitz investors. This section investigates whether usual Markowitz investors would choose to invest in BPT and BPT_{CPT} optimal portfolios.

In the canonical MVT problem, investors minimize the objective function $(1/2)V(R)$ subject to the constraint $E[R] = e$, where e stands for the expected return set by the agent. Each level of e corresponds to a portfolio on the MV frontier. The greater the level of e, the riskier the portfolio will be. As the BPT optimal portfolio is located in the upper right part of the MV frontier, the MV investor whose optimal portfolio coincides with the BPT portfolio would deliberately choose to set a high level for e in her optimization program.

Another way to solve the MVT problem is to maximize $E(R) - (\gamma/2)V(R)$ with a different level of γ, where γ stands for the risk aversion coefficient. Each level of γ corresponds to a portfolio on the MV frontier. The less the investor is risk averse, the smaller her risk aversion coefficient will be, and the higher the expected return of her optimal portfolio will be. As the BPT optimal portfolio is associated with a significant couple expected return/risk, only a MV investor characterized by a small risk aversion coefficient will choose this kind of portfolio. In this section, we investigate the level of the risk aversion coefficient that corresponds to the location of the BPT optimal portfolio on the MV efficient frontier. Remember that a MV investor maximizes

$$X'\overline{R} - (\gamma/2)X'VX \text{ u.c. } X'1 = 1, \tag{17}$$

where X is the vector of portfolio weights of the 80 stocks, $\overline{R}$ is the vector of the 80 expected returns, V is the return covariance, and $1 = [1, 1, \ldots.1]'$.

[20] This result for the BPT and BPT_{CPT} optimal portfolios is robust to changes in both aspiration and α levels.

[21] Fig. 3 describes the following specification: α is equal to 0.3, an aspiration level that corresponds to the long-term risk-free rate and no short-sales. The graph is similar when taking other specifications.

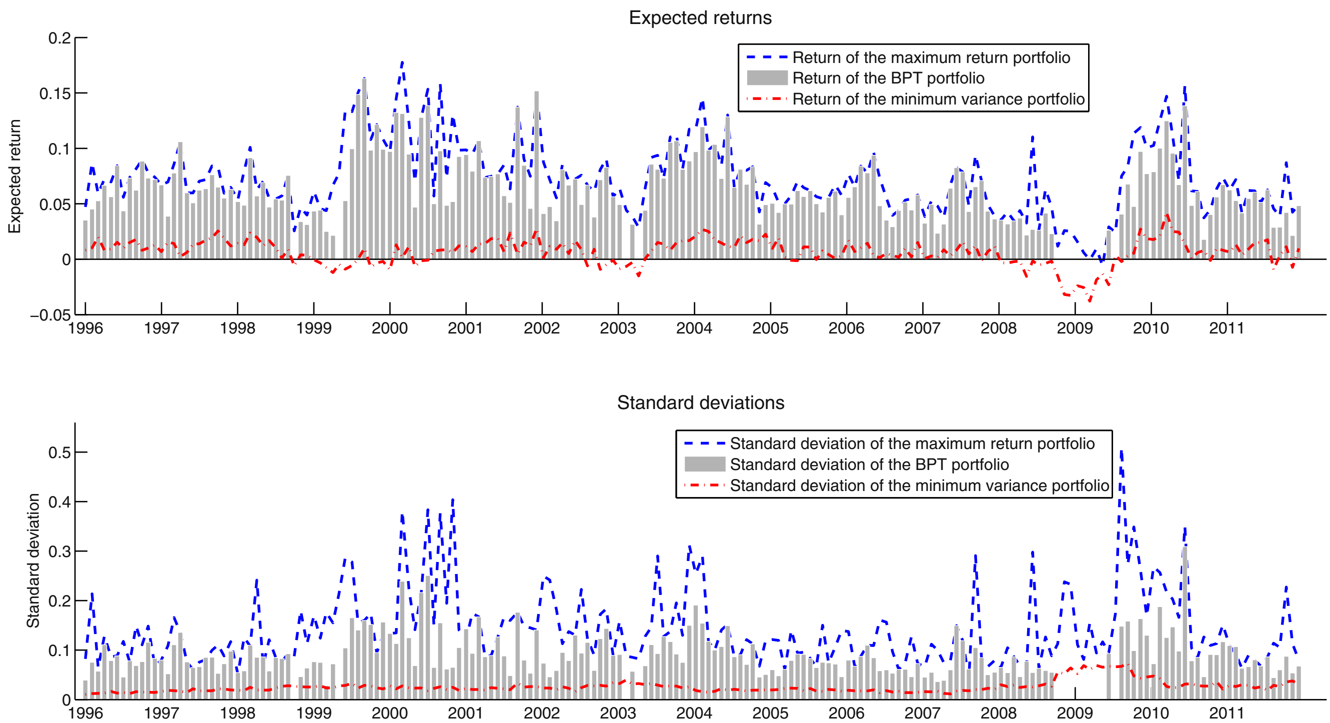

Fig. 3. Expected return and standard deviation of the BPT optimal portfolio.

The closed form solution of this optimization problem is

$$X^* = \frac{1}{\gamma}V^{-1}\left[\overline{R} - \left(\frac{1'V^{-1}\overline{R} - \gamma}{1'V^{-1}1}\right)1\right]. \tag{18}$$

X^* gives the composition of the efficient portfolio for a given level of γ. Furthermore, we know that X^* is also the well-known solution to the canonical MVT problem, where investors minimize the objective function $(1/2)X'VX$ under the constraints $X'\overline{R} = e$ and $X'1 = 1$. Therefore, X^* can also be written as (see Huang and Litzenberger, 1988)

$$X^* = X_1 + e \times X_2, \tag{19}$$

with[22]

$$\begin{aligned} X_1 &= \frac{1}{D}\left[BV^{-1}1 - AV^{-1}\overline{R}\right] \\ X_2 &= \frac{1}{D}\left[CV^{-1}\overline{R} - AV^{-1}1\right]. \end{aligned} \tag{20}$$

Using Eq. (19), we have

$$\begin{aligned} X_1 + e \times X_2 &= \frac{1}{\gamma}\left[V^{-1}\overline{R} - \frac{A}{C} \times V^{-1}1\right] + \frac{V^{-1}1}{C} \\ \Rightarrow e \times X_2 &= \frac{1}{\gamma}\left[V^{-1}\overline{R} - \frac{A}{C} \times V^{-1}1\right] + \frac{V^{-1}1}{C} - X_1. \end{aligned} \tag{21}$$

It is then possible to show (see proof in 9) that

$$e \times X_2 = \left(\frac{1}{\gamma} + \frac{A}{D}\right)\left(V^{-1}\overline{R} - \frac{A}{C} \times V^{-1}1\right). \tag{22}$$

Therefore, the expected return of the portfolio e and the risk aversion coefficient γ are linked by the following hyperbolic relation:

$$e = \frac{D}{C}\left(\frac{1}{\gamma} + \frac{A}{D}\right) \Leftrightarrow \gamma = \frac{D}{e \times C - A}. \tag{23}$$

Our aim here is to discuss the value of the risk aversion coefficient associated with the expected return of the BPT and $\mathrm{BPT_{CPT}}$ portfolios (γ_{BPT} and γ_{CPT}). We compute the relative value of γ_{BPT} and γ_{CPT} by comparing γ_{BPT} and γ_{CPT} to the risk aversion coefficient of typical MV portfolios. Our goal is to determine to what extent the BPT investors could be less risk averse than typical MV investors. To this end, we consider two categories of MV portfolios: (1) a set of 4 efficient portfolios with expected return equal to the expected return of the minimum-variance (min-v) portfolio + 2 % (respectively + 5 %, + 10 % and + 20 %), and (2) the optimal portfolio of an investor that seeks to reach the S&P 500 return.[23] We then compute the coefficient of risk aversion γ associated with these different portfolios. Tables 4 and 5 display the six ratios γ_{MV} on γ_{BPT} and γ_{CPT}. These ratios indicate how much lower is the risk aversion of a MV investor who chooses the BPT portfolio compared to typical MV investors. For instance, when short-sales are allowed and $\alpha = 0.2$, the MV investor who chooses to invest in the BPT portfolio is 10 times less risk averse than a MV investor who chooses the efficient portfolio associated with the S&P 500 expected return.

For all the specifications tested, the risk aversion coefficient γ_{BPT} (respectively, γ_{CPT}) associated with the BPT optimal portfolio ($\mathrm{BPT_{CPT}}$ optimal portfolio) is significantly lower than the risk aversion coefficient γ_{MV} associated with usual MV portfolios.[24] The MV investor who decides to invest in the BPT portfolio has a lower degree of risk aversion than typical MV investors. This result is reinforced when short-sales are allowed. Thus, even if the BPT optimal portfolio is mean variance

[22] with

$$\begin{aligned} A &= 1'V^{-1}\overline{R}, \quad B = \overline{R}'V^{-1}\overline{R} \\ C &= 1'V^{-1}1, \quad D = BC - A^2 \end{aligned}.$$

[23] For several dates in our sample, the expected return of the S&P 500 is too small to obtain a portfolio located on the efficient frontier. We exclude these dates from the calculation of $\gamma_{S\&P}$.

[24] We use a Jarque–Bera and a Kolmogorov–Smirnov test to show that the series of risk aversion coefficients are normally distributed. The significance of the difference between means is then assessed using a Student t-test.

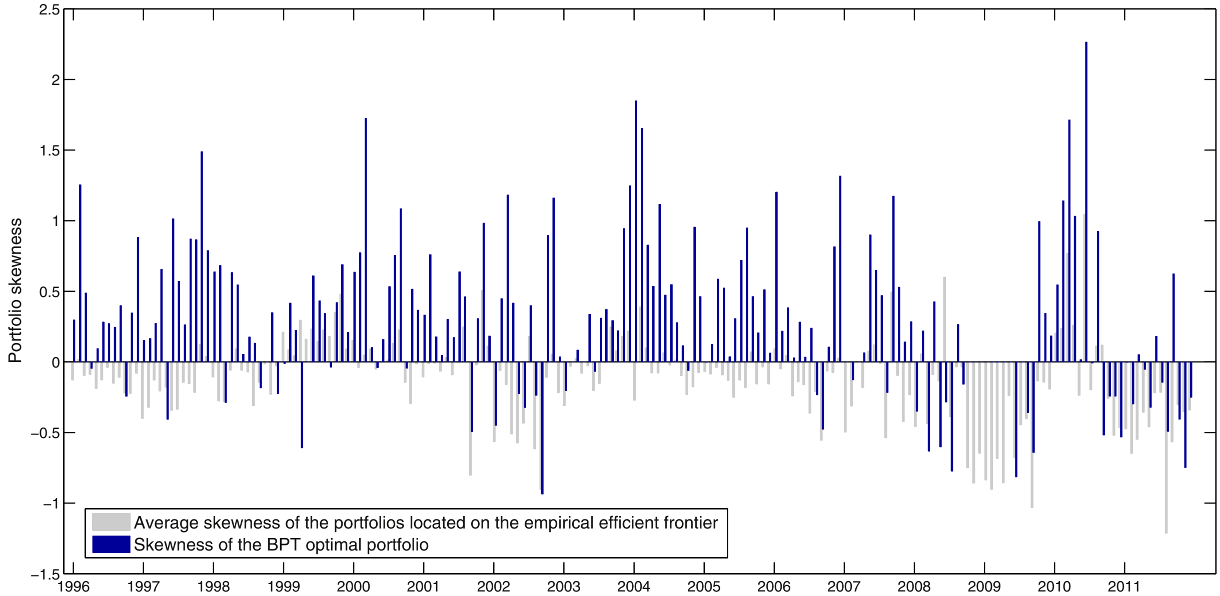

Fig. 4. Skewness of the BPT optimal portfolio.

efficient, it will not be chosen by ordinary MV investors. Therefore, the coincidence of the BPT optimal portfolio and the MV efficient set does not mean that these two theories lead to the same asset allocation.

6. Robustness tests

6.1. Portfolio weights and stock selection

In the baseline approach, any stock among the 1452 in our sample has the same probability of being included in the portfolio. However, in practice, investors tend to favor large capitalization stocks. In order to test whether market capitalization may impact our results, we run our entire analysis using two alternative processes to include stocks in the portfolio. The first alternative selection process is quite simplistic and consists of retaining only the 80 largest capitalization stocks in our sample. Results (unreported) are unchanged. The second selection process consists of introducing a tilt toward large capitalization stocks. Our selection rule is as follows: For each date t, we build ten deciles of capitalization. The probability of being included in the portfolio is therefore a function of the capitalization deciles. For instance, we set the probability so that a stock in the first decile of capitalization (largest ones) is ten times more likely to be selected in the portfolio compared to a stock in the tenth decile. Similarly, a stock in the second decile of capitalization is nine times more likely to be selected, and so on. Tables C.1 to C.5 present the empirical results using this alternative selection process (these tables correspond to Tables 1 to 5). This confirms that our results are not driven by the selection process.

6.2. BPT_{CPT} with alternative reference point κ

In our study, we set the reference point κ equal to the long-term risk-free rate (i.e., the 10-year U.S. Treasury Bond). However, the return of the stock market is also another natural reference point for constructing the utility over gains and losses. We therefore replicate our analysis of

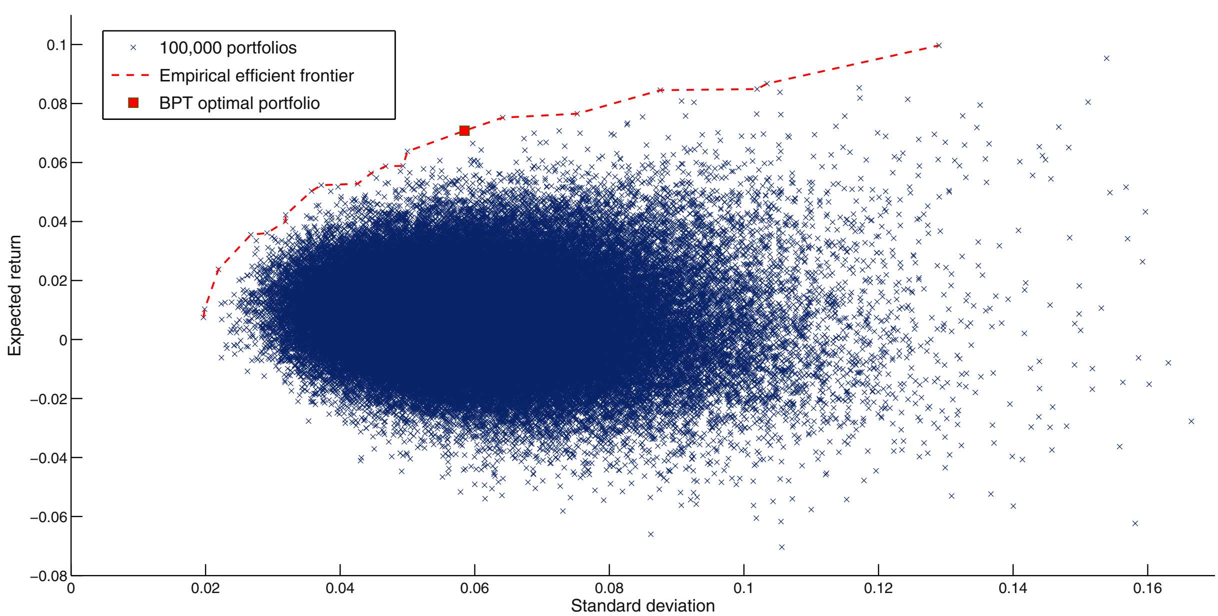

Fig. 5. Illustration of the empirical efficient frontier — BPT optimal portfolio located on the upper right part of the frontier.

Table 4
BPT optimal portfolio: risk aversion coefficient.

	$\gamma_{mv+2\%}/\gamma_{BPT}$	$\gamma_{mv+5\%}/\gamma_{BPT}$	$\gamma_{mv+10\%}/\gamma_{BPT}$	$\gamma_{mv+20\%}/\gamma_{BPT}$	$\gamma_{S\&P}/\gamma_{BPT}$	$\gamma_{mv+2\%}/\gamma_{BPT}$	$\gamma_{mv+5\%}/\gamma_{BPT}$	$\gamma_{mv+10\%}/\gamma_{BPT}$	$\gamma_{mv+20\%}/\gamma_{BPT}$	$\gamma_{S\&P}/\gamma_{BPT}$
	Panel A: Short sales forbidden					Panel B: Short sales allowed				
Aspiration level $= r_{ST}$										
$\alpha = 0.1$	32.2996	13.0937	6.6898	3.4845	7.5437	48.3701	19.6084	10.0183	5.2182	9.7672
$\alpha = 0.2$	34.1458	13.8421	7.0722	3.6836	7.6432	55.9851	22.6954	11.5955	6.0397	10.7102
$\alpha = 0.3$	36.4884	14.7918	7.5574	3.9364	7.4394	58.4194	23.6822	12.0997	6.3023	11.3327
Aspiration level $= r_{LT}$										
$\alpha = 0.1$	32.6953	13.2541	6.7718	3.5272	7.945	48.1962	19.5379	9.9823	5.1994	9.8848
$\alpha = 0.2$	34.3457	13.9232	7.1136	3.7052	7.6931	55.9023	22.6619	11.5783	6.0307	10.7406
$\alpha = 0.3$	36.6276	14.8482	7.5862	3.9514	7.4756	58.4196	23.6823	12.0997	6.3023	11.3386
Aspiration level $= r_{S\&P}$										
$\alpha = 0.1$	32.6224	13.2246	6.7567	3.5193	8.5501	47.9591	19.4418	9.9332	5.1738	10.8804
$\alpha = 0.2$	33.1758	13.4489	6.8713	3.579	7.6037	55.7659	22.6066	11.5501	6.016	10.9065
$\alpha = 0.3$	35.8799	14.5451	7.4313	3.8707	7.4061	58.5279	23.7262	12.1221	6.314	11.3991
Aspiration level $= r_{1\%}$										
$\alpha = 0.1$	31.3987	12.7285	6.5032	3.3873	6.8878	48.518	19.6684	10.0489	5.2341	9.6168
$\alpha = 0.2$	34.3707	13.9333	7.1188	3.7079	7.6012	56.2522	22.8037	11.6508	6.0685	11.011
$\alpha = 0.3$	36.6031	14.8383	7.5811	3.9487	7.4548	58.4085	23.6778	12.0974	6.3011	11.3457
Aspiration level $= r_{5\%}$										
$\alpha = 0.1$	32.3501	13.1142	6.7003	3.4899	7.5952	48.3907	19.6168	10.0225	5.2204	9.933
$\alpha = 0.2$	34.4775	13.9766	7.1409	3.7194	7.7473	55.9326	22.6741	11.5846	6.034	10.7478
$\alpha = 0.3$	36.6779	14.8686	7.5966	3.9568	7.4788	58.3944	23.6721	12.0945	6.2996	11.3257
Aspiration level $= r_{10\%}$										
$\alpha = 0.1$	33.7376	13.6767	6.9876	3.6396	9.0455	49.0683	19.8915	10.1629	5.2935	10.2115
$\alpha = 0.2$	35.091	14.2253	7.2679	3.7856	7.5001	55.7324	22.593	11.5431	6.0124	10.8163
$\alpha = 0.3$	36.8559	14.9408	7.6335	3.976	7.5744	58.4719	23.7035	12.1105	6.3079	11.5013

This table compares the risk aversion coefficient of the BPT optimal portfolio to the risk aversion coefficients of other portfolios such as the minimum variance portfolio or the S&P 500 portfolio. The probability of failure (α) to reach the aspiration level takes the value 0.1, 0.2 and 0.3. The different values for the aspiration level are: (1) the long-term risk-free rate (r_{ST}); (2) the long-term risk-free rate (r_{LT}); (3) the S&P 500 return over the past 3 years ($r_{S\&P}$); (4) an annualized rate of 1% ($r_{1\%}$); (5) an annualized rate of 5% ($r_{5\%}$); and, (6) an annualized rate of 10% ($r_{10\%}$). γ_{BPT} is the risk aversion coefficient of the BPT optimal portfolio. γ_{mv} is the risk aversion coefficient of the MV efficient portfolio with an expected return that is equal to the minimum variance portfolio return, plus an annualized return of x%. $\gamma_{S\&P}$ represents the risk aversion coefficient of the MV efficient portfolio with an expected return that is equal to the of the S&P 500 return.

the BPT_{CPT} portfolio using this alternative reference point (the S&P 500 return over the previous three years). Tables C.6, C.7 and C.8 present the empirical results using this alternative reference point. Our conclusions remain unchanged.

7. Conclusion

This study aims to empirically select the optimal portfolio of the Behavioral Portfolio Theory (BPT) developed by Shefrin and Statman

Table 5
BPT_{CPT} optimal portfolio: risk aversion coefficient.

	$\gamma_{mv+2\%}/\gamma_{CPT}$	$\gamma_{mv+5\%}/\gamma_{CPT}$	$\gamma_{mv+10\%}/\gamma_{CPT}$	$\gamma_{mv+20\%}/\gamma_{CPT}$	$\gamma_{S\&P}/\gamma_{CPT}$	$\gamma_{mv+2\%}/\gamma_{CPT}$	$\gamma_{mv+5\%}/\gamma_{CPT}$	$\gamma_{mv+10\%}/\gamma_{CPT}$	$\gamma_{mv+20\%}/\gamma_{CPT}$	$\gamma_{S\&P}/\gamma_{CPT}$
	Panel A: Short sales forbidden					Panel B: Short sales allowed				
Aspiration level $= r_{ST}$										
$\alpha = 0.1$	31.9941	12.9699	6.6265	3.4515	7.5433	47.9916	19.455	9.9399	5.1773	9.6163
$\alpha = 0.2$	32.3978	13.1335	6.7101	3.4951	7.2409	53.7829	21.8027	11.1394	5.8021	10.0211
$\alpha = 0.3$	32.9833	13.3709	6.8314	3.5582	6.7932	54.9865	22.2906	11.3886	5.9319	10.4282
Aspiration level $= r_{LT}$										
$\alpha = 0.1$	32.3875	13.1293	6.708	3.494	7.9084	47.7906	19.3735	9.8983	5.1556	9.717
$\alpha = 0.2$	32.6663	13.2424	6.7658	3.524	7.2696	53.7316	21.7819	11.1287	5.7966	10.0275
$\alpha = 0.3$	33.1203	13.4264	6.8598	3.573	6.8378	54.9993	22.2958	11.3913	5.9333	10.4438
Aspiration level $= r_{S\&P}$										
$\alpha = 0.1$	32.4121	13.1393	6.7131	3.4966	8.5342	47.5005	19.2559	9.8382	5.1243	10.6873
$\alpha = 0.2$	31.5653	12.796	6.5377	3.4053	7.1831	53.5618	21.7131	11.0936	5.7782	10.347
$\alpha = 0.3$	32.486	13.1693	6.7284	3.5046	6.717	55.0464	22.3149	11.4011	5.9384	10.4383
Aspiration level $= r_{1\%}$										
$\alpha = 0.1$	31.0483	12.5865	6.4306	3.3495	6.8604	48.1358	19.5134	9.9698	5.1929	9.4776
$\alpha = 0.2$	32.5341	13.1888	6.7384	3.5098	7.1625	53.9567	21.8731	11.1754	5.8208	10.3155
$\alpha = 0.3$	33.0379	13.393	6.8427	3.5641	6.819	55.0176	22.3032	11.3951	5.9353	10.4347
Aspiration level $= r_{5\%}$										
$\alpha = 0.1$	32.0348	12.9864	6.635	3.4559	7.5551	48.01	19.4624	9.9437	5.1793	9.7701
$\alpha = 0.2$	32.7841	13.2901	6.7902	3.5367	7.3296	53.7404	21.7854	11.1306	5.7975	10.0246
$\alpha = 0.3$	33.178	13.4498	6.8717	3.5792	6.8598	54.9979	22.2952	11.391	5.9332	10.4765
Aspiration level $= r_{10\%}$										
$\alpha = 0.1$	33.472	13.569	6.9326	3.6109	9.0088	48.6202	19.7098	10.0701	5.2451	10.1019
$\alpha = 0.2$	33.4949	13.5783	6.9374	3.6134	7.2535	53.6014	21.7291	11.1018	5.7825	10.1343
$\alpha = 0.3$	33.5175	13.5874	6.9421	3.6159	6.9609	55.0371	22.3111	11.3991	5.9374	10.5281

This table compares the risk aversion coefficient of the BPT_{CPT} optimal portfolio to the risk aversion coefficients of other portfolios such as the minimum variance portfolio or the S&P 500 portfolio. The probability of failure (α) to reach the aspiration level takes the value 0.1, 0.2 and 0.3. The different values for the aspiration level are: (1) the long-term risk-free rate (r_{ST}); (2) the long-term risk-free rate (r_{LT}); (3) the S&P 500 return over the past 3 years ($r_{S\&P}$); (4) an annualized rate of 1% ($r_{1\%}$); (5) an annualized rate of 5% ($r_{5\%}$); and, (6) an annualized rate of 10% ($r_{10\%}$). γ_{CPT} is the risk aversion coefficient of the BPT_{CPT} optimal portfolio. γ_{mv} is the risk aversion coefficient of the MV efficient portfolio with an expected return that is equal to the minimum variance portfolio return, plus an annualized return of x%. $\gamma_{S\&P}$ represents the risk aversion coefficient of the MV efficient portfolio with an expected return that is equal to the of the S&P 500 return.

(2000). We compare the BPT portfolio to portfolios chosen by Markowitz investors. Simulations are run using U.S. stock prices from the CRSP database for the 1995–2011 period. We show that in 70% of cases, the BPT optimal portfolio is located on the MV efficient frontier. This echoes recent studies (Das et al., 2004; Levy and Levy, 2004; Levy et al., 2012) that also underline the coincidence of MVT and BPT-like models. The new contribution of our study lies in the fact that we empirically compare the asset allocations generated by BPT and MVT without restrictions. We do not make any assumption about the distribution of returns, allow for short sales and take all the features of BPT into consideration. This study not only shows that the BPT optimal portfolio tends to be efficient, but also provides empirical evidence that this portfolio is always characterized by a high level of risk, high returns and a positive skewness. Our results also indicate the absence of the BPT optimal portfolio during periods of financial crises. The BPT portfolio is therefore chosen by investors who exhibit risk-seeking behavior and are attracted by potential high gains. However, when potential losses are too high, the safety-first constraint leads investors to refrain from selecting any portfolio. This result underlines a weakness of the BPT model. Do investors really exit the market when the safety-first constraint is not met? This result warrants further extended research. Finally, we provide empirical evidence that efficient BPT portfolios are always located on the upper right of the MV frontier. Therefore, even if the BPT optimal portfolio is often located on the MV frontier, it will not be chosen by typical MV investors since it is associated with an extremely low degree of risk aversion. It follows that MVT and BPT do not lead to the same asset allocation: MV investors with usual levels of risk aversion would not invest in the BPT optimal portfolio.

Appendix A. Cumulative prospect theory

The Cumulative Prospect Theory (CPT) is based on four main observations.

(1) Investors use decision weights instead of probabilities, and overweight probabilities of extreme events.
(2) As in the Expected Utility Theory, investors determine the subjective value of each outcome via a value function. However, under CPT, utility is derived from changes in wealth, relative to a reference point with respect to which gains and losses are defined.
(3) Sensitivity with respect to the reference point mentioned in (2) decreases and individuals are loss averse.
(4) Experimental evidence has established a fourfold pattern of risk attitudes: risk aversion for most gains and low probability losses, and risk seeking for most losses and low probability gains.

Under CPT, a prospect $X = ((x_i, p_i), i = -m, \dots n)$ is evaluated through a valuation function. This function is defined as follows:

$$V(X) = V\left(X^+\right) + V\left(X^-\right), \tag{A.1}$$

where $X^+ = \max(X; 0)$ and $X^- = \min(X; 0)$.

We set

$$V\left(X^+\right) = \sum_{i=1}^{n-1}\left(w^+\left(\sum_{j=i}^{n} p_j\right) - w^+\left(\sum_{j=i+1}^{n} p_j\right)v(x_i) + w^+(p_n)v(x_n)\right) \dots V\left(X^-\right) = \sum_{i=m+1}^{0}\left(w^-\left(\sum_{j=-m}^{i} p_j\right) - w^-\left(\sum_{j=-m}^{i-1} p_j\right)v(x_i) + w^-(p_{-m})v(x_{-m})\right) \tag{A.2}$$

where v is a strictly increasing value function defined with respect to a reference point x_0 satisfying $v(x_0) = v(0) = 0$, and with $w^+(0) = 0 = w^-(0)$ and $w^+(1) = 1 = w^-(1)$.

The functional form for the value function v, proposed by Tversky and Kahneman (1992), is given by

$$v(x) = \left\{\begin{array}{ll} x^\alpha & \text{if } x > 0 \\ -\lambda(-x)^\beta & \text{if } x < 0 \end{array}\right\}. \tag{A.3}$$

For $0 < \alpha < 1$ and $0 < \beta < 1$ the value function v is concave over gains and convex over losses. The parameter λ determines the degree of loss aversion (Köbberling and Wakker, 2005). Based on experimental evidence, Tversky and Kahneman (1992) estimated the values of the parameters α, β and λ. They found $\alpha = \beta = 0.88$ and $\lambda = 2.25$.

Tversky and Kahneman (1992) proposed the following functional form for the weighting function w

$$w^+(p) = \frac{p^{\gamma+}}{\left[p^{\gamma+} + (1-p)^{\gamma+}\right]^{1/\gamma+}} \tag{A.4}$$

$$w^-(p) = \frac{p^{\gamma-}}{\left[p^{\gamma-} + (1-p)^{\gamma-}\right]^{1/\gamma-}}. \tag{A.5}$$

For $\gamma < 1$, this form integrates the overweighting of low probabilities and the greater sensitivity to changes in probabilities for extremely low and extremely high probabilities. The weighting function is concave near 0 and convex near 1. Tversky and Kahneman (1992) estimated the parameters γ^+ and γ^- as 0.61 and 0.69 respectively.

Appendix B. Proof of Eq. (22)

Eq. (22) gives

$$e\times X_2=\frac{1}{\gamma}\left[V^{-1}\overline{R}-\frac{A}{C}\times V^{-1}1\right]+\frac{V^{-1}1}{C}-X_1.$$

Since $\frac{V^{-1}1}{C}$ is independent from e and γ, $\frac{V^{-1}1}{C}-X_1$ is proportional to $V^{-1}\overline{R}-\frac{A}{C}\times V^{-1}1$.

Proof. We have

$$\frac{V^{-1}1}{C}-X_1=\frac{V^{-1}1}{C}-\frac{1}{D}\left[BV^{-1}1-AV^{-1}\overline{R}\right]=V^{-1}1\left[\frac{1}{C}-\frac{B}{D}\right]+\frac{A}{D}V^{-1}\overline{R}=V^{-1}1\left[\frac{D-BC}{CD}\right]+\frac{A}{D}V^{-1}\overline{R}=-V^{-1}1\frac{A^2}{CD}+\frac{A}{D}V^{-1}\overline{R}=\frac{A}{D}\left[V^{-1}\overline{R}-\frac{A}{C}V^{-1}1\right].$$

It follows that Eq. (22) can be written as

$$e\times X_2=\left(\frac{1}{\gamma}+\frac{A}{D}\right)\left(V^{-1}\overline{R}-\frac{A}{C}\times V^{-1}1\right).$$

Appendix C. Robustness tests — tables

Table C.1
Stock selection tilted toward large capitalization stocks — proportion of simulations for which the BPT optimal portfolio is MV efficient.

Aspiration level	r_{ST}	r_{LT}	$r_{S\&P}$	$r_{1\%}$	$r_{5\%}$	$r_{10\%}$
Panel A: Short sales forbidden						
$\alpha=0.1$	0.7180	0.7228	0.7049	0.7139	0.7280	0.7124
$\alpha=0.2$	0.7452	0.7464	0.7418	0.7298	0.7407	0.7342
$\alpha=0.3$	0.7338	0.7376	0.7304	0.7261	0.7337	0.7393
Panel B: Short sales allowed						
$\alpha=0.1$	0.7592	0.7628	0.7641	0.7409	0.7619	0.7956
$\alpha=0.2$	0.7151	0.7191	0.7228	0.7068	0.7162	0.7339
$\alpha=0.3$	0.6658	0.6672	0.6654	0.6608	0.6678	0.6793

For this table, the stock selection process was tilted in favor of large capitalization stocks. This table provides the proportion of simulations for which the BPT optimal portfolio is MV efficient. The probability of failure (α) to reach the aspiration level takes the value 0.1, 0.2 and 0.3. The different values for the aspiration level are: (1) the long-term risk-free rate (r_{ST}); (2) the long-term risk-free rate (r_{LT}); (3) the S&P 500 return over the past 3 years ($r_{S\&P}$); (4) an annualized rate of 1% ($r_{1\%}$); (5) an annualized rate of 5% ($r_{5\%}$); and, (6) an annualized rate of 10% ($r_{10\%}$). The total number of simulations is 4,012. For each simulation, the BPT optimal portfolio is selected from a sample of 100,000 portfolios.

Table C.2
Stock selection tilted toward large capitalization stocks — proportion of simulations for which the BPT_{CPT} optimal portfolio is MV efficient.

Aspiration level	r_{ST}	r_{LT}	$r_{S\&P}$	$r_{1\%}$	$r_{5\%}$	$r_{10\%}$
Panel A: Short sales forbidden						
$\alpha=0.1$	0.7460	0.7510	0.7371	0.7362	0.7628	0.7582
$\alpha=0.2$	0.7826	0.7803	0.7912	0.7750	0.7787	0.7652
$\alpha=0.3$	0.7952	0.7979	0.7894	0.7950	0.7958	0.7893
Panel B: Short sales allowed						
$\alpha=0.1$	0.787	0.7948	0.7866	0.7736	0.7944	0.8149
$\alpha=0.2$	0.7721	0.7714	0.779	0.7713	0.7701	0.7771
$\alpha=0.3$	0.7615	0.7614	0.7622	0.7602	0.7619	0.7605

For this table, the stock selection process was tilted in favor of large capitalization stocks. This table provides the proportion of simulations for which the BPT_{CPT} optimal portfolio is MV efficient. The probability of failure (α) to reach the aspiration level takes the value 0.1, 0.2 and 0.3. The different values for the aspiration level are: (1) the long-term risk-free rate (r_{ST}); (2) the long-term risk-free rate (r_{LT}); (3) the S&P 500 return over the past 3 years ($r_{S\&P}$); (4) an annualized rate of 1% ($r_{1\%}$); (5) an annualized rate of 5% ($r_{5\%}$); and, (6) an annualized rate of 10% ($r_{10\%}$). The total number of simulations is 4,012. For each simulation, the BPT_{CPT} optimal portfolio is selected from a sample of 100,000 portfolios.

Table C.3
Stock selection tilted toward large capitalization stocks — proportion of simulations for which the BPT optimal portfolio is the same portfolio as the BPT_{CPT} optimal portfolio.

Aspiration level	r_{ST}	r_{LT}	$r_{S\&P}$	$r_{1\%}$	$r_{5\%}$	$r_{10\%}$
Panel A: Short sales forbidden						
$\alpha = 0.1$	0.8917	0.9017	0.8772	0.8793	0.907	0.8807
$\alpha = 0.2$	0.6851	0.7098	0.7006	0.6682	0.6989	0.7303
$\alpha = 0.3$	0.4938	0.497	0.5032	0.4832	0.4981	0.5203
Panel B: Short sales allowed						
$\alpha = 0.1$	0.8836	0.8881	0.8941	0.871	0.8836	0.9092
$\alpha = 0.2$	0.6658	0.6751	0.6695	0.6533	0.6721	0.6926
$\alpha = 0.3$	0.5202	0.5259	0.5148	0.5165	0.5245	0.5393

For this table, the stock selection process was tilted in favor of large capitalization stocks. This table provides the proportion of simulations for which the BPT optimal portfolio is identical to the BPT_{CPT} optimal portfolio. The probability of failure (α) to reach the aspiration level takes the value 0.1, 0.2 and 0.3. The different values for the aspiration level are: (1) the long-term risk-free rate (r_{ST}); (2) the long-term risk-free rate (r_{LT}); (3) the S&P 500 return over the past 3 years ($r_{S\&P}$) (4) an annualized rate of 1% ($r_{1\%}$); (5) an annualized rate of 5% ($r_{5\%}$); and, (6) an annualized rate of 10% ($r_{10\%}$). The total number of simulations is 4,012. For each simulation, the BPT and the BPT_{CPT} optimal portfolios are selected from a sample of 100,000 portfolios.

Table C.4
Stock selection tilted toward large capitalization stocks — BPT optimal portfolio: risk aversion coefficient.

	$\gamma_{mv+2\%}/\gamma_{BPT}$	$\gamma_{mv+5\%}/\gamma_{BPT}$	$\gamma_{mv+10\%}/\gamma_{BPT}$	$\gamma_{mv+20\%}/\gamma_{BPT}$	$\gamma_{S\&P}/\gamma_{BPT}$	$\gamma_{mv+2\%}/\gamma_{BPT}$	$\gamma_{mv+5\%}/\gamma_{BPT}$	$\gamma_{mv+10\%}/\gamma_{BPT}$	$\gamma_{mv+20\%}/\gamma_{BPT}$	$\gamma_{S\&P}/\gamma_{BPT}$
	Panel A: Short sales forbidden					Panel B: Short sales allowed				
Aspiration level = r_{ST}										
$\alpha = 0.1$	30.8631	12.5114	6.3923	3.3295	8.1246	44.2333	17.9314	9.1615	4.7719	19.5799
$\alpha = 0.2$	32.6765	13.2465	6.7679	3.5251	11.3398	50.7531	20.5745	10.5118	5.4752	18.1548
$\alpha = 0.3$	35.298	14.3092	7.3108	3.8079	11.6683	52.9164	21.4514	10.9599	5.7086	17.3212
Aspiration level = r_{LT}										
$\alpha = 0.1$	31.3068	12.6912	6.4842	3.3774	8.3886	44.4365	18.0138	9.2036	4.7938	20.3019
$\alpha = 0.2$	32.8917	13.3337	6.8124	3.5483	12.2952	50.682	20.5457	10.4971	5.4676	17.785
$\alpha = 0.3$	35.3986	14.35	7.3317	3.8188	11.7273	52.8518	21.4252	10.9465	5.7016	17.334
Aspiration level = $r_{S\&P}$										
$\alpha = 0.1$	30.3832	12.3168	6.2929	3.2777	9.3899	42.6077	17.2725	8.8248	4.5965	20.7391
$\alpha = 0.2$	31.6261	12.8207	6.5503	3.4118	10.2474	50.2005	20.3504	10.3974	5.4156	17.4414
$\alpha = 0.3$	34.1092	13.8273	7.0646	3.6797	11.4164	53.0464	21.5041	10.9868	5.7226	17.4549
Aspiration level = $r_{1\%}$										
$\alpha = 0.1$	29.8715	12.1094	6.1869	3.2225	7.9733	44.3594	17.9825	9.1876	4.7855	18.8621
$\alpha = 0.2$	32.8571	13.3197	6.8053	3.5446	12.1462	50.9266	20.6448	10.5478	5.4939	18.2075
$\alpha = 0.3$	35.3136	14.3155	7.3141	3.8096	11.6322	52.9308	21.4573	10.9629	5.7102	17.3321
Aspiration level = $r_{5\%}$										
$\alpha = 0.1$	31.126	12.6180	6.4467	3.3579	8.3672	44.6015	18.0807	9.2377	4.8116	20.4758
$\alpha = 0.2$	32.9852	13.3717	6.8318	3.5584	12.3758	50.6484	20.5320	10.4902	5.4639	17.793
$\alpha = 0.3$	35.4063	14.3531	7.3333	3.8196	11.7551	52.8736	21.4341	10.9510	5.7040	17.3486
Aspiration level = $r_{10\%}$										
$\alpha = 0.1$	33.5932	13.6181	6.9577	3.6240	10.5249	45.6579	18.5089	9.4565	4.9256	24.0178
$\alpha = 0.2$	33.7202	13.6696	6.9840	3.6377	12.1787	50.5340	20.4856	10.4665	5.4516	18.2019
$\alpha = 0.3$	35.5365	14.4059	7.3602	3.8337	11.8259	52.9117	21.4495	10.9589	5.7081	17.5394

For this table, the stock selection process was tilted in favor of large capitalization stocks. This table compares the risk aversion coefficient of the BPT optimal portfolio to the risk aversion coefficients of other portfolios such as the minimum variance portfolio or the S&P 500 portfolio. The probability of failure (α) to reach the aspiration level takes the value 0.1, 0.2 and 0.3. The different values for the aspiration level are: (1) the long-term risk-free rate (r_{ST}); (2) the long-term risk-free rate (r_{LT}); (3) the S&P 500 return over the past 3 years ($r_{S\&P}$); (4) an annualized rate of 1% ($r_{1\%}$); (5) an annualized rate of 5% ($r_{5\%}$); and, (6) an annualized rate of 10% ($r_{10\%}$). γ_{BPT} is the risk aversion coefficient of the BPT optimal portfolio. γ_{mv} is the risk aversion coefficient of the MV efficient portfolio with an expected return that is equal to the minimum variance portfolio return, plus an annualized return of x%. $\gamma_{S\&P}$represents the risk aversion coefficient of the MV efficient portfolio with an expected return that is equal to the of the S&P 500 return.

Table C.5
Stock selection tilted toward large capitalization stocks — BPT_{CPT} optimal portfolio: risk aversion coefficient.

	$\gamma_{mv+2\%}/\gamma_{CPT}$	$\gamma_{mv+5\%}/\gamma_{CPT}$	$\gamma_{mv+10\%}/\gamma_{CPT}$	$\gamma_{mv+20\%}/\gamma_{CPT}$	$\gamma_{S\&P}/\gamma_{CPT}$	$\gamma_{mv+2\%}/\gamma_{CPT}$	$\gamma_{mv+5\%}/\gamma_{CPT}$	$\gamma_{mv+10\%}/\gamma_{CPT}$	$\gamma_{mv+20\%}/\gamma_{CPT}$	$\gamma_{S\&P}/\gamma_{CPT}$
	Panel A: Short sales forbidden					Panel B: Short sales allowed				
Aspiration level = r_{ST}										
$\alpha = 0.1$	30.5982	12.404	6.3374	3.3009	7.9835	43.823	17.7651	9.0765	4.7276	19.5175
$\alpha = 0.2$	31.0086	12.5704	6.4224	3.3452	10.9814	48.5446	19.6792	10.0544	5.237	17.3922
$\alpha = 0.3$	31.769	12.8786	6.5799	3.4272	10.9102	49.368	20.013	10.225	5.3258	16.5193

Table C.5 (*continued*)

	$\gamma_{mv+2\%}/\gamma_{CPT}$	$\gamma_{mv+5\%}/\gamma_{CPT}$	$\gamma_{mv+10\%}/\gamma_{CPT}$	$\gamma_{mv+20\%}/\gamma_{CPT}$	$\gamma_{S\&P}/\gamma_{CPT}$	$\gamma_{mv+2\%}/\gamma_{CPT}$	$\gamma_{mv+5\%}/\gamma_{CPT}$	$\gamma_{mv+10\%}/\gamma_{CPT}$	$\gamma_{mv+20\%}/\gamma_{CPT}$	$\gamma_{S\&P}/\gamma_{CPT}$
	Panel A: Short sales forbidden					Panel B: Short sales allowed				
Aspiration level $= r_{LT}$										
$\alpha = 0.1$	31.0912	12.6039	6.4395	3.3541	8.2434	44.0349	17.851	9.1204	4.7505	20.2352
$\alpha = 0.2$	31.299	12.6881	6.4826	3.3765	11.9583	48.5162	19.6677	10.0485	5.2339	17.0001
$\alpha = 0.3$	31.898	12.9309	6.6066	3.4412	10.9857	49.3097	19.9893	10.2129	5.3195	16.5213
Aspiration level $= r_{S\&P}$										
$\alpha = 0.1$	30.0595	12.1856	6.2258	3.2428	8.6535	42.1849	17.101	8.7372	4.5509	20.694
$\alpha = 0.2$	30.124	12.2117	6.2392	3.2498	10.5765	48.0259	19.4689	9.947	5.181	16.8638
$\alpha = 0.3$	30.7121	12.4502	6.361	3.3132	10.6974	49.4157	20.0323	10.2348	5.3309	16.4933
Aspiration level $= r_{1\%}$										
$\alpha = 0.1$	29.5807	11.9915	6.1267	3.1912	7.9288	43.9104	17.8005	9.0946	4.737	18.7479
$\alpha = 0.2$	31.11	12.6115	6.4434	3.3561	11.5133	48.6276	19.7128	10.0716	5.2459	17.4267
$\alpha = 0.3$	31.7289	12.8624	6.5716	3.4229	10.8717	49.3978	20.025	10.2311	5.329	16.5216
Aspiration level $= r_{5\%}$										
$\alpha = 0.1$	30.9347	12.5404	6.4071	3.3372	8.2303	44.1994	17.9177	9.1545	4.7682	20.411
$\alpha = 0.2$	31.3737	12.7184	6.498	3.3846	12.0512	48.4642	19.6466	10.0378	5.2283	17.0186
$\alpha = 0.3$	31.9263	12.9424	6.6125	3.4442	10.9932	49.3282	19.9968	10.2167	5.3215	16.5493
Aspiration level $= r_{10\%}$										
$\alpha = 0.1$	33.2325	13.4719	6.883	3.5851	10.4508	45.2958	18.3621	9.3815	4.8865	23.9635
$\alpha = 0.2$	32.2648	13.0796	6.6826	3.4807	11.0331	48.4125	19.6256	10.0271	5.2227	17.4329
$\alpha = 0.3$	32.1673	13.0401	6.6624	3.4702	11.0276	49.3682	20.0131	10.225	5.3258	16.6054

For this table, the stock selection process was tilted in favor of large capitalization stocks. This table compares the risk aversion coefficient of the BPT_{CPT} optimal portfolio to the risk aversion coefficients of other portfolios such as the minimum variance portfolio or the S&P 500 portfolio. The probability of failure (α) to reach the aspiration level takes the value 0.1, 0.2 and 0.3. The different values for the aspiration level are: (1) the long-term risk-free rate (r_{ST}); (2) the long-term risk-free rate (r_{LT}); (3) the S&P 500 return over the past 3 years ($r_{S\&P}$); (4) an annualized rate of 1% ($r_{1\%}$); (5) an annualized rate of 5% ($r_{5\%}$); and, (6) an annualized rate of 10% ($r_{10\%}$). γ_{CPT} is the risk aversion coefficient of the BPT_{CPT} optimal portfolio. γ_{mv} is the risk aversion coefficient of the MV efficient portfolio with an expected return that is equal to the minimum variance portfolio return, plus an annualized return of x%. $\gamma_{S\&P}$ represents the risk aversion coefficient of the MV efficient portfolio with an expected return that is equal to the of the S&P 500 return.

Table C.6
Alternative reference point κ (S&P 500 return) - Proportion of simulations for which the BPT_{CPT} optimal portfolio is MV efficient.

Aspiration level	r_{ST}	r_{LT}	$r_{S\&P}$	$r_{1\%}$	$r_{5\%}$	$r_{10\%}$
Panel A: Short sales forbidden						
$\alpha = 0.1$	0.7601	0.7593	0.7434	0.7664	0.762	0.7519
$\alpha = 0.2$	0.7823	0.7714	0.7728	0.7773	0.7743	0.768
$\alpha = 0.3$	0.7775	0.7865	0.7746	0.7837	0.7883	0.7876
Panel B: Short sales allowed						
$\alpha = 0.1$	0.7939	0.7888	0.8133	0.7927	0.7911	0.8256
$\alpha = 0.2$	0.7805	0.782	0.78	0.7823	0.7825	0.7919
$\alpha = 0.3$	0.7591	0.7609	0.7628	0.7571	0.7603	0.7642

In this table, we use the S&P 500 return over the past 3 years as reference point for co $r_{1\%}$ nstructing the utility over gain and losses. This table provides the proportion of simulations for which the BPT_{CPT} optimal portfolio is MV efficient. The probability of failure (α) to reach the aspiration level takes the value 0.1, 0.2 and 0.3. The different values for the aspiration level are: (1) the long-term risk-free rate (r_{ST}); (2) the long-term risk-free rate (r_{LT}); (3) the S&P 500 return over the past 3 years ($r_{S\&P}$); (4) an annualized rate of 1% ($r_{1\%}$); (5) an annualized rate of 5% ($r_{5\%}$); and, (6) an annualized rate of 10% ($r_{10\%}$). The total number of simulations is 4012. For each simulation, the BPT_{CPT} optimal portfolio is selected from a sample of 100,000 portfolios.

Table C.7
Alternative reference point κ (S&P 500 return) — proportion of simulations for which the BPT optimal portfolio is the same portfolio as the BPT_{CPT} optimal portfolio.

Aspiration level	r_{ST}	r_{LT}	$r_{S\&P}$	$r_{1\%}$	$r_{5\%}$	r10%
Panel A: Short sales forbidden						
$\alpha = 0.1$	0.8896	0.8956	0.8872	0.8745	0.8898	0.9132
$\alpha = 0.2$	0.7069	0.7178	0.7218	0.6864	0.712	0.7486
$\alpha = 0.3$	0.5042	0.5071	0.516	0.4931	0.5115	0.5403
Panel B: Short sales allowed						
$\alpha = 0.1$	0.9033	0.9012	0.8956	0.8884	0.9061	0.9108
$\alpha = 0.2$	0.6728	0.6786	0.6838	0.6625	0.675	0.7
$\alpha = 0.3$	0.5328	0.5381	0.5326	0.5311	0.539	0.5513

In this table, we use the S&P 500 return over the past 3 years as reference point for constructing the utility over gain and losses. This table provides the proportion of simulations for which the BPT optimal portfolio is the same portfolio as the BPT_{CPT} optimal portfolio. The probability of failure (α) to reach the aspiration level takes the value 0.1, 0.2 and 0.3. The different values for the aspiration level are: (1) the long-term risk-free rate (r_{ST}); (2) the long-term risk-free rate (r_{LT}); (3) the S&P 500 return over the past 3 years ($r_{S\&P}$) (4) an annualized rate of 1% ($r_{1\%}$); (5) an annualized rate of 5% ($r_{5\%}$); and, (6) an annualized rate of 10% ($r_{10\%}$). The total number of simulations is 4012. For each simulation, the BPT and the BPT_{CPT} optimal portfolios are selected from a sample of 100,000 portfolios.

Table C.8
Alternative reference point κ (S&P 500 return) — BPT_{CPT} optimal portfolio: risk aversion coefficient.

	$\gamma_{mv+2\%}/\gamma_{CPT}$	$\gamma_{mv+5\%}/\gamma_{CPT}$	$\gamma_{mv+10\%}/\gamma_{CPT}$	$\gamma_{mv+20\%}/\gamma_{CPT}$	$\gamma_{S\&P}/\gamma_{CPT}$	$\gamma_{mv+2\%}/\gamma_{CPT}$	$\gamma_{mv+5\%}/\gamma_{CPT}$	$\gamma_{mv+10\%}/\gamma_{CPT}$	$\gamma_{mv+20\%}/\gamma_{CPT}$	$\gamma_{S\&P}/\gamma_{CPT}$
	Panel A: Short sales forbidden					Panel B: Short sales allowed				
Aspiration level = r_{ST}										
$\alpha = 0.1$	31.9819	12.9649	6.624	3.4502	7.4898	47.9909	19.4547	9.9397	5.1772	9.5991
$\alpha = 0.2$	32.48	13.1668	6.7272	3.5039	7.2504	53.7495	21.7891	11.1324	5.7985	10.0123
$\alpha = 0.3$	33.0455	13.3961	6.8443	3.5649	6.7961	54.9332	22.269	11.3776	5.9262	10.4219
Aspiration level = r_{LT}										
$\alpha = 0.1$	32.3919	13.1311	6.7089	3.4944	7.908	47.7848	19.3711	9.897	5.155	9.6966
$\alpha = 0.2$	32.7368	13.271	6.7804	3.5316	7.2777	53.6996	21.7689	11.1221	5.7931	10.0239
$\alpha = 0.3$	33.1864	13.4532	6.8735	3.5801	6.8418	54.944	22.2734	11.3798	5.9273	10.4198
Aspiration level = $r_{S\&P}$										
$\alpha = 0.1$	32.2859	13.0882	6.687	3.483	8.4038	47.4583	19.2388	9.8294	5.1198	10.6823
$\alpha = 0.2$	31.5528	12.791	6.5351	3.4039	7.1899	53.5257	21.6984	11.0861	5.7743	10.3498
$\alpha = 0.3$	32.5265	13.1857	6.7368	3.509	6.7163	55.014	22.3017	11.3943	5.9349	10.4336
Aspiration level = $r_{1\%}$										
$\alpha = 0.1$	31.0647	12.5931	6.434	3.3513	6.8178	48.1349	19.5131	9.9696	5.1928	9.4638
$\alpha = 0.2$	32.613	13.2208	6.7547	3.5183	7.1703	53.9133	21.8556	11.1664	5.8162	10.3062
$\alpha = 0.3$	33.1101	13.4223	6.8577	3.5719	6.8258	54.963	22.2811	11.3838	5.9294	10.4288
Aspiration level = $r_{5\%}$										
$\alpha = 0.1$	32.0306	12.9847	6.6341	3.4555	7.5274	48.0031	19.4597	9.9423	5.1786	9.7507
$\alpha = 0.2$	32.8543	13.3186	6.8047	3.5443	7.3367	53.711	21.7736	11.1245	5.7943	10.0211
$\alpha = 0.3$	33.2495	13.4788	6.8865	3.5869	6.8641	54.934	22.2693	11.3778	5.9263	10.4521
Aspiration level = $r_{10\%}$										
$\alpha = 0.1$	33.4624	13.5651	6.9306	3.6099	9.0086	48.6202	19.7098	10.0701	5.2451	10.0831
$\alpha = 0.2$	33.5524	13.6016	6.9493	3.6196	7.2496	53.5961	21.727	11.1007	5.7819	10.137
$ss = 0.3$	33.563	13.6059	6.9515	3.6208	6.9318	54.9907	22.2923	11.3895	5.9324	10.5045

In this table, we use the S&P 500 return over the past 3 years as reference point for constructing the utility over gain and losses. This table compares the risk aversion coefficient of the BPT_{CPT} optimal portfolio to the risk aversion coefficients of other portfolios such as the minimum variance portfolio or the S&P 500 portfolio. The probability of failure (α) to reach the aspiration level takes the value 0.1, 0.2 and 0.3. The different values for the aspiration level are: (1) the long-term risk-free rate (r_{ST}); (2) the long-term risk-free rate (r_{LT}); (3) the S&P 500 return over the past 3 years ($r_{S\&P}$); (4) an annualized rate of 1% ($r_{1\%}$); (5) an annualized rate of 5% ($r_{5\%}$); and, (6) an annualized rate of 10% ($r_{10\%}$). γ_{CPT} is the risk aversion coefficient of the BPT_{CPT} optimal portfolio. γ_{mv} is the risk aversion coefficient of the MV efficient portfolio with an expected return that is equal to the minimum variance portfolio return, plus an annualized return of x%. $\gamma_{S\&P}$ represents the risk aversion coefficient of the MV efficient portfolio with an expected return that is equal to the of the S&P 500 return.

References

Alexander, G.J., Baptista, A.M., 2011. Portfolio selection with mental accounts and delegation. J. Bank. Financ. 35, 2637–2656.

Allais, M., 1953. Le comportement de l'homme rationnel devant le risque: critique des postulats et axiomes de l'ecole americaine. Econometrica 21, 503–546.

Ameur, B.H., Prigent, J.L., 2013. Optimal portfolio positioning under ambiguity. Econ. Model. 34, 89–97.

Bali, T.G., Cakici, N., Whitelaw, R.F., 2011. Maxing out: stocks as lotteries and the cross-section of expected returns. J. Financ. Econ. 99, 427–446.

Baptista, A.M., 2012. Portfolio selection with mental accounts and background risk. J. Bank. Financ. 36, 968–980.

Cox, J.C., Huang, C., 1989. Optimal consumption and portfolio policies when asset prices follow a diffusion process. J. Econ. Theory 49, 33–83.

Das, S.R., Statman, M., 2013. Options and structured products in behavioral portfolios. J. Econ. Dyn. Control. 37, 137–153.

Das, S.R., Markowitz, H., Statman, M., 2004. Portfolio optimization with mental accounts. J. Financ. Quant. Anal. 45, 311–334.

Erev, I., Ert, E., Yechiam, E., 2008. Loss aversion, diminishing sensitivity, and the effect of experience on repeated decisions. J. Behav. Decis. Mak. 21, 575–597.

Hens, T., Mayor, J., 2014. Cumulative Prospect Theory and Mean Variance Analysis: A Rigorous Comparison. Swiss Finance Institute Research Paper, pp. 14–23.

Huang, C.F., Litzenberger, R., 1988. Foundations of Financial Economics (North-Holland).

Hull, J., White, A., 1998. Incorporating volatility updating into the historical simulation method for value-at-risk. Manag. Sci. 52, 558–566.

Jiang, C., Ma, Y., An, Y., 2013. International portfolio selection with exchange rate risk: a behavioural portfolio theory perspective. J. Bank. Financ. 37, 648–659.

Kahneman, D., Tversky, A., 1979. Prospect theory: an analysis of decision under risk. Econometrica 47, 263–291.

Köbberling, V., Wakker, P., 2005. An index of loss aversion. J. Econ. Theory 122, 119–131.

Kumar, A., 2009. Who gambles in the stock market? J. Financ. 64, 1889–1933.

Levy, M., Levy, H., 2004. Prospect theory and mean variance analysis. Rev. Financ. Stud. 17, 1015–1041.

Levy, H., De Giorgi, E., Hens, T., 2012. Two paradigms and nobel prizes in economics: a contradiction or coexistence? Eur. Financ. Manag. 18, 163–182.

Li, D., Ng, W.L., 2000. Optimal dynamic portfolio selection: multi-period mean-variance formulation. Math. Financ. 10, 387–406.

Lopes, L., 1987. Between hope and fear: the psychology of risk. Adv. Exp. Soc. Psychol. 20, 255–295.

Mandelbrot, B., 1963. The variation of certain speculative prices. J. Bus. 36, 394–419.

Mantegna, R., Stanley, H., 1995. Scaling behavior in the dynamics of an economic index. Nature 376, 46–49.

Markowitz, H., 1952. Portfolio selection. J. Financ. 7, 77–91.

Merton, R.C., 1971. Optimum consumption and portfolio rules in a continuous-time model. J. Econ. Theory 3, 373–413.

Quiggin, J., 1982. A theory of anticipated utility. J. Econ. Behav. Organ. 3, 323–343.

Rademacher, H., 1937. On the partition function p(n). Proc. Lond. Math. Soc. 43, 241–254.

Roy, A.D., 1952. Safety-first and the holding of assets. Econometrica 20, 431–449.

Shefrin, H., Statman, M., 2000. Behavioral portfolio theory. J. Financ. Quant. Anal. 35, 127–151.

Telser, L., 1955. Safety first and hedging. Rev. Econ. Stud. 23, 1–16.
Thaler, R.H., 1985. Mental accounting and consumer choice. Mark. Sci. 27, 15–25.
Thaler, R.H., 1999. Mental accounting matters. J. Behav. Decis. Mak. 12, 183–206.
Tversky, A., Kahneman, D., 1991. Loss aversion in riskless choice: a reference dependent model. Q. J. Econ. 106, 1039–1061.
Tversky, A., Kahneman, D., 1992. Advances in prospect theory: cumulative representation of uncertainty. J. Risk Uncertain. 5, 297–323.
Yao, H., Li, Z., Chen, S., 2014. Continuous-time mean-variance portfolio selection with only risky assets. Econ. Model. 36, 244–251.